GANDHI ACADEMY OF TECHNOLOGY AND ENGINEERING

GOLANTHARA, BERHAMPUR

LECTURE NOTES

ON

ENGINEERING PHYSICS

For 2ND Semester

(As per Syllabus prescribed by SCTE&VT, Odisha)

LECTURE NOTES

ON

PHYSICS

For

Diploma Students

(According to SCTE & VT Syllabus)

Prepared by

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PREFACE

Engineering and technology would not exist without physics. Any technology involving energy (heat, light, sound, electromagnetic, mechanical),involves physics! All manufactured items originate in physics-based technology.

In Engineering Physics you will learn about the physics concepts that form the foundation of any mechanical design. To be an engineer, you not only have to be able to come up with the ideas, but you must also understand the physics behind those ideas so that you can design and manufacture a successful product.

This study material is designed to provide students in Pure & Applied Science who wish to study engineering or physical sciences at university with an enhanced background in order to improve their chances of success in their chosen program. The material will be presented using the normal mix of lectures and problem-solving sessions. Some useful data and information are given in the appendices.

I hope that the study material will prove helpful and will meet the needs of the students at undergraduate level.

Suggestions and criticisms for further improvement of the study material are most welcome.

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Th.2a. Engineering Physics
(1^{ST} / 2^{ND} Sem Common)

Theory: 4 Periods per Week | **I.A:** 20 Marks
Total Periods: 60 Periods | **End Sem Exam :** 80 Marks
Examination: 3 Hours | **TOTAL MARKS :** 100 Marks

UNIT 1 - UNITS AND DIMENSIONS

1.1 Physical quantities - (Definition).
1.2 Definition of fundamental and derived units, systems of units (FPS, CGS, MKS and SI units).
1.3 Definition of dimension and Dimensional formulae of physical quantities.
1.4 Dimensional equations and Principle of homogeneity.
1.5 Checking the dimensional correctness of Physical relations.

UNIT 2 - SCALARS AND VECTORS

2.1 Scalar and Vector quantities (definition and concept), Representation of a Vector – examples, types of vectors.
2.2 Triangle and Parallelogram law of vector Addition (Statement only). Simple Numerical.
2.3 Resolution of Vectors – Simple Numericals on Horizontal and Vertical components.
2.4 Vector multiplication (scalar product and vector product of vectors).

UNIT 3 - KINEMATICS

3.1 Concept of Rest and Motion.
3.2 Displacement, Speed, Velocity, Acceleration & FORCE (Definition, formula, dimension & SI units).
3.3 Equations of Motion under Gravity (upward and downward motion) - no derivation.
3.4 Circular motion: Angular displacement, Angular velocity and Angular acceleration (definition, formula & SI units).
3.5 Relation between –(i) Linear & Angular velocity, (ii) Linear & Angular acceleration).
3.6 Define Projectile, Examples of Projectile.
3.7 Expression for Equation of Trajectory, Time of Flight, Maximum Height and Horizontal Range for a projectile fired at an angle, Condition for maximum Horizontal Range.

UNIT 4 – WORK AND FRICTION

4.1 Work – Definition, Formula & SI units.
4.2 Friction – Definition & Concept.
4.3 Types of friction (static, dynamic), Limiting Friction (Definition with Concept).
4.4 Laws of Limiting Friction (Only statement, No Experimental Verification).
4.5 Coefficient of Friction – Definition & Formula, Simple Numericals.
4.6 Methods to reduce friction.

UNIT 5 - GRAVITATION

5.1 Newton's Laws of Gravitation – Statement and Explanation.
5.2 Universal Gravitational Constant (G)- Definition, Unit and Dimension.
5.3 Acceleration due to gravity (g)- Definition and Concept.
5.4 Definition of mass and weight.
5.5 Relation between g and G.
5.6 Variation of g with altitude and depth (No derivation – Only Explanation).
5.7 Kepler's Laws of Planetary Motion (Statement only).

UNIT 6 - OSCILLATIONS AND WAVES

6.1 Simple Harmonic Motion (SHM) - Definition & Examples.
6.2 Expression (Formula/Equation) for displacement, velocity, acceleration of a body/ particle in SHM.
6.3. Wave motion – Definition & Concept.
6.4 Transverse and Longitudinal wave motion – Definition, Examples & Comparison.
6.5 Definition of different wave parameters (Amplitude, Wavelength, Frequency, Time Period.
6.6 Derivation of Relation between Velocity, Frequency and Wavelength of a wave
6.7 Ultrasonics – Definition, Properties & Applications.

UNIT 7 - HEAT AND THERMODYNAMICS

7.1 Heat and Temperature – Definition & Difference
7.2 Units of Heat (FPS, CGS, MKS & SI).
7.3 Specific Heat (concept, definition, unit, dimension and simple numerical)
7.4 Change of state (concept), Latent Heat (concept, definition, unit, dimension and simple

numerical)
7.5 Thermal Expansion – Definition & Concept
7.6 Expansion of Solids (Concept)
7.7 Coefficient of linear, superficial and cubical expansions of Solids – Definition & Units.
7.8 Relation between **α**, **β** & **ϒ**
7.9 Work and Heat - Concept & Relation.
7.10 Joule's Mechanical Equivalent of Heat (Definition, Unit)
7.11 First Law of Thermodynamics (Statement and concept only)

UNIT 8 – OPTICS

8.1 Reflection & Refraction – Definition.
8.2 Laws of reflection and refraction (Statement only)
8.3 Refractive index – Definition, Formula &Simple numerical.
8.4 Critical Angle and Total internal reflection – Concept, Definition & Explanation
8.5 Refraction through Prism (Ray Diagram & Formula only – NO derivation)..
8.6 Fiber Optics – Definition, Properties & Applications.

UNIT 9 – ELECTROSTATICS & MAGNETOSTATICS

9.1 Electrostatics – Definition & Concept.
9.2 Statement & Explanation of Coulombs laws, Definition of Unit charge.
9.3 Absolute & Relative Permittivity (**ε**) – Definition, Relation & Unit.
9.4 Electric potential and Electric Potential difference (Definition, Formula & SI Units).
9.5 Electric field, Electric field intensity (E) – Definition, Formula & Unit.
9.6 Capacitance - Definition, Formula & Unit.
9.7 Series and Parallel combination of Capacitors (No derivation, Formula for effective/Combined/total capacitance & Simple numericals).
9.8 Magnet, Properties of a magnet.
9.9 Coulomb's Laws in Magnetism – Statement & Explanation, Unit Pole (Definition).
9.10 Magnetic field, Magnetic Field intensity (H) - (Definition, Formula & SI Unit).
9.11 Magnetic lines of force (Definition and Properties)
9.12 Magnetic Flux (**Φ**) & Magnetic Flux Density (B) – Definition, Formula & Unit.

UNIT 10 – CURRENT ELECTRICITY

10.1 Electric Current – Definition, Formula & SI Units.
10.2 Ohm's law and its applications.
10.3 Series and Parallel combination of resistors (No derivation, Formula for effective/ Combined/ total resistance & Simple numericals).
10.4 Kirchhoff's laws (Statement & Explanation with diagram).
10.5 Application of Kirchhoff's laws to Wheatstone bridge - Balanced condition of Wheatstone's Bridge – Condition of Balance (Equation).

UNIT 11 – ELECTROMAGNETISM & ELECTROMAGNETIC INDUCTION

11.1 Electromagnetism – Definition & Concept.
11.2 Force acting on a current carrying conductor placed in a uniform magnetic field, Fleming's Left Hand Rule
11.3 Faraday's Laws of Electromagnetic Induction (Statement only)
11.4 Lenz's Law (Statement)
11.5 Fleming's Right Hand Rule
11.6 Comparison between Fleming's Right Hand Rule and Fleming's Left Hand Rule.

UNIT 12 - MODERN PHYSICS

12.1 LASER & laser beam (Concept and Definition)
12.2 Principle of LASER (Population Inversion & Optical Pumping)
12.3 Properties & Applications of LASER
12.4 Wireless Transmission – Ground Waves, Sky Waves, Space Waves (Concept & Definition)

RECOMMENDED BOOKS

1. Text Book of Physics for Class XI (Part-I, Part-II) N.C.E.R.T
2. Text Book of Physics for Class XII (Part-I, Part-II) N.C.E.R.T
3. Text Book of Engineering Physics by Barik, Das, Sharma, Kalyani Publisher
4. Concepts in Physics by H. C. Verma, Vol. I & II, Bharti Bhawan Ltd. New Delhi

A BRIEF INTRODUCTION OF ENGINEERING PHYSICS

Physics is the study of the mechanical universe. It is the basic science that underlies all the natural sciences. It is a search for the basic rules of the behavior of matter and energy on every scale: from the interaction of subatomic particles, to the motion of everyday objects, to the evolution of galaxies. Physics consists of many sub-fields, including particle and nuclear physics, atomic and molecular spectroscopy, optics, solid state physics, biological and medical physics, computational physics, acoustics, astrophysics and cosmology.

Engineering physics is the study of the combined disciplines of physics, engineering and mathematics in order to develop an understanding of the interrelationships of these three disciplines. Fundamental physics is combined with problem solving and engineering skills, which then has broad applications. Career paths for Engineering physics is usually (broadly) "Engineering, applied science or applied physics through research, teaching or entrepreneurial engineering". This interdisciplinary knowledge is designed for the continuous innovation occurring with technology.

Unlike traditional engineering disciplines, engineering science/physics is not necessarily confined to a particular branch of science or physics. Instead, engineering science/physics is meant to provide a more thorough grounding in applied physics for a selected specialty such as optics, quantum physics, materials science, applied mechanics, nanotechnology, micro fabrication, mechanical engineering, electrical engineering, biophysics, control theory, aerodynamics, energy, solid-state physics, etc. It is the discipline devoted to creating and optimizing engineering solutions through enhanced understanding and integrated application of mathematical, scientific, statistical, and engineering principles.

It is a bridge between pure and applied science, utilizing fundamental concepts in today's rapidly changing and highly technical engineering environment. An engineering physicist is motivated by the application of science for advancing technology and sustainability. The program emphasizes the solid foundations of modern scientific principles, mathematical rigor, technical know-how in designing, building and doing experiments, the knowledge essential for a successful professional career in science and technology. The program is recommended for students interested in newly developing areas of physics, modern technology, instrumentation, and experimentation. It also enriches a student with analytical skills of mathematics and scientific reasoning; technical skills of design, construction and operation of systems including nanotechnology, space instrumentation, particle accelerators and more; leadership skills as engineering physicists are called to manage projects involving electrical, mechanical or chemical components and tasks. They tend to be versatile and adaptable to the projects as they evolve.

Undergraduate program in engineering science focuses on the creation and use of more advanced experimental or computational techniques where standard approaches are inadequate (i.e., development of engineering solutions to contemporary problems in the physical and life sciences by applying fundamental principles).The study of Engineering Physics emphasizes the application of basic scientific principles to the design of equipment, which includes electronic and electro-mechanical systems, for use in measurements, communications, and data acquisition.

The program is recommended for students interested in newly developing areas of physics, high technology, instrumentation and communications. Our program is fully accredited by the Canadian Engineering Accreditation Board so graduates will be eligible to be certified as a professional engineer. Graduates are also qualified for entry into graduate schools in Physics or other disciplines.

Engineering Physics Educational Outcomes:

CO_1- To understand and apply the concepts of physical quantities, units, and dimensions, including dimensional analysis and the principle of homogeneity. They will also solve problems related to scalars, vectors, and kinematics, including motion, projectile trajectories, and circular motion.

CO_2- To Explain the concepts of work, friction, and gravitation, including types of friction, the laws of limiting friction, and the coefficient of friction. They will also gain knowledge of gravitation, Kepler's laws, oscillations, wave motion, and ultrasonics, and be able to apply these concepts to solve related problems.

CO_3- To apply the concepts of heat, temperature, thermal expansion, and thermodynamics, including specific heat, latent heat, and the first law of thermodynamics. They will also learn the principles of optics, including reflection, refraction, refractive index, total internal reflection, and fiber optics, and be able to solve related numerical problems.

CO_4- To analyse the fundamental concepts of electrostatics and magnetostatics, including Coulomb's law, electric potential, electric fields, capacitance, and magnetic properties. They will also learn to solve problems involving the series and parallel combinations of capacitors and resistors, Ohm’s law, and Kirchhoff's laws, and apply these concepts in real-world electrical circuit analysis.

CO_5- To describe the concepts of electromagnetism and electromagnetic induction, including the force on a current-carrying conductor, Fleming’s Left and Right Hand Rules, and Faraday’s and Lenz’s laws. They will also gain knowledge of lasers, their principles, properties, applications, and wireless transmission types, including ground, sky, and space waves.

Engineering Physics offers a wide range of exciting opportunities for students who are curious about the way things work and who want to use their talents to make the world a better place. Engineers are inventors and problem-solvers. They use science and technology to find faster, better, and cheaper ways of doing things. They take ideas and raw materials and design machinery and systems that increase efficiency and productivity. They develop new products to simplify household tasks. They find new energy sources and ways to protect the environment. Almost everything we use today has been designed and produced by engineers.

Discoveries by physicists, like quantum phenomena and the theory of the Big Bang, have literally transformed have literally transformed our view of the natural world. Inventions like the transistor and the laser have fuelled the modern technological revolution. We can look forward to even more exhilarating breakthroughs in the future - a future that holds exciting opportunities for the physics students of today.

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TABLE 1.3

S. No.	Physical quantity	Relation	Dimensional formula	Units in SI
1.	Pressure	$\frac{\text{force}}{\text{area}}$	$\frac{[MLT^{-2}]}{[L^2]} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
2.	Impulse	force × time	$[MLT^{-2} \times T] = [MLT^{-1}]$	Ns
3.	Power	$\frac{\text{work}}{\text{time}}$	$\frac{[ML^2 T^{-2}]}{[T]} = [M^1 L^2 T^{-3}]$	Js^{-1} or W
4.	Velocity of light	velocity	$[M^0 L^1 T^{-1}]$	ms^{-1}
5.	Frequency	$\frac{1}{\text{time period}}$	$\frac{1}{[T]} = T^{-1}$	s^{-1} or Hz
6.	Velocity gradient	$\frac{\text{velocity}}{\text{distance}}$	$\frac{[M^0 L^1 T^{-1}]}{[L]} = M^0 L^0 T^{-1}$	s^{-1}
7.	Wavelength	$\frac{\text{velocity}}{\text{frequency}}$	$\frac{[M^0 L^1 T^{-1}]}{[T^{-1}]} = M^0 L^1 T^0$	m
8.	Gravitation constant (G)	$\frac{\text{force} \times (\text{distance})^2}{(\text{mass})^2}$	$\frac{[MLT^{-2} \times L^2]}{[M^2]} = [M^{-1} L^3 T^{-2}]$	$Nm^2\ kg^{-2}$
9.	Planck's constant (h)	$\frac{\text{energy}}{\text{frequency}}$	$\frac{[ML^2 T^{-2}]}{[T^{-1}]} = [M^1 L^2 T^{-1}]$	Js
10.	Strain	$\frac{\text{change in length}}{\text{original length}}$	$[M^0 L^0 T^0]$ (dimensionless)	No units
11.	Stress	$\frac{\text{force}}{\text{area}}$	$\frac{[MLT^{-2}]}{[L^2]} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
12.	Rate of flow	$\frac{\text{volume}}{\text{time}}$	$\frac{[M^0 L^3 T^0]}{[T^1]} = M^0 L^3 T^{-1}$	$m^3\ s^{-1}$
13.	Co-efficient of elasticity	$\frac{\text{stress}}{\text{strain}}$	$\frac{[M^1 L^{-1} T^{-2}]}{[M^0 L^0 T^0]} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
14.	Co-efficient of viscosity	$\frac{\text{force}}{\text{area} \times \text{velocity gradient}}$	$\frac{[MLT^{-2}]}{[L^2 \times T^{-1}]} = [M^1 L^{-1} T^{-1}]$	$Nm^{-2}\ s$
15.	Surface tension	$\frac{\text{force}}{\text{length}}$	$\frac{[MLT^{-2}]}{[L]} = [M^1 L^0 T^{-2}]$	Nm^{-1}
16.	Surface energy	$\frac{\text{energy}}{\text{area}}$	$\frac{[ML^2 T^{-2}]}{[L^2]} = [M^1 L^0 T^{-2}]$	Jm^{-2}
17.	Angle	$\frac{\text{length of arc}}{\text{radius}}$	$\frac{[L]}{[L]} = [M^0 L^0 T^0]$ (dimensionless)	No units
18.	Angular velocity	$\frac{\text{angle}}{\text{time}}$	$\frac{[M^0 L^0 T^0]}{[T]} = [M^0 L^0 T^{-1}]$	s^{-1}
19.	Angular acceleration	$\frac{\text{angular velocity}}{\text{time}}$	$\frac{[M^0 L^0 T^{-1}]}{[T]} = [M^0 L^0 T^{-2}]$	s^{-2}
20.	Torque	force × distance	$[MLT^{-2} \times L] = [M^1 L^2 T^{-2}]$	Nm
21.	Angular momentum	momentum × distance	$[MLT^{-1} \times L] = [M^1 L^2 T^{-1}]$	$kg\ m^2\ s^{-1}$

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22.	Moment of inertia	mass × (radius)^2	$[ML^2] = [M^1 L^2 T^0]$	*kg m*^{2}
23.	Radius of gyration	distance	$[M^0 L^1 T^0]$	*m*
24.	Coefficient of friction	$\frac{\text{force}}{\text{normal reaction}}$	$\frac{[MLT^{-2}]}{[MLT^{-2}]} = [M^0 L^0 T^0]$	No units
25.	Temperature	Fundamental quantity	$[M^0 L^0 T^0 K^1]$	*K*
26.	Heat	Energy	$[M^1 L^2 T^{-2}]$	*J*
27.	Gas constant (R)	$\frac{\text{pressure} \times \text{volume}}{\text{temperature}}$	$\frac{[M^1 L^{-1} T^{-2}] \times L^3}{[K^1]} = [M^1L^2T^{-2}K^{-1}]$	JK^{-1}
28.	Boltzmann constant (k)	$\frac{R}{N}$	$\frac{[M^1 L^2 T^{-2} K^{-1}]}{[M^0 L^0 T^0]} = [M^1L^2T^{-2}K^{-1}]$	JK^{-1}
29.	Coefficient of thermal conductivity	$\frac{Qd}{A\,(\theta_2 - \theta_1)\,t}$	$\frac{[M^1 L^2 T^{-2}] \times L}{[L^2] \times [K^1] \times [T^1]} = [M^1 L^1 T^{-3} K^{-1}]$	$Wm^{-1} K^{-1}$
30.	Electric current (i)	Fundamental quantity	$[M^0 L^0 T^0 A^1]$	*A*
31.	Potential difference	$\frac{\text{Work}}{\text{charge}}$	$\frac{[M^1 L^2 T^{-2}]}{[A^1 T^1]} = [M^1 L^2 T^{-3} A^{-1}]$	*V*
32.	Resistance	$\frac{\text{Potential difference}}{\text{current}}$	$\frac{[M^1 L^2 T^{-3} A^{-1}]}{[A]} = [M^1 L^2 T^{-3} A^{-2}]$	*ohm*
33.	Resistivity (ρ)	$\frac{\text{Resistance} \times \text{Area}}{\text{Length}}$	$\frac{[M^1L^2T^{-3}A^{-2}] \times [L^2]}{[L^1]} = [M^1L^3T^{-3}A^{-2}]$	Ωm
34.	Capacitance (C)	$\frac{\text{Charge}}{\text{Potential difference}}$	$\frac{[A^1T^1]}{[M^1L^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$	*F*
35.	Permittivity (ε)	$\frac{(\text{Charge})^2}{\text{Force} \times (\text{distance})^2}$	$\frac{[A^1T^1]^2}{[M^1L^1T^{-2}] \times [L^1]^2} = [M^{-1}L^{-3}T^4A^2]$	Fm^{-1}
36.	Magnetic flux density (B)	$\frac{\text{Force}}{\text{Charge} \times \text{Velocity}}$	$\frac{[M^1L^1T^{-2}]}{[A^1T^1] \times [L^1T^{-1}]} = [M^1L^0T^{-2}A^{-1}]$	$NA^{-1}m^{-1}$ or $Wb\ m^{-2}$
37.	Magnetic pole (m)	$\frac{\text{Force}}{\text{magnetic flux density}}$	$\frac{[M^1L^1T^{-2}]}{[M^1L^0T^{-2}A^{-1}]} = [L^1A^1]$	*Am*
38.	Magnetic permeability (μ)	$\frac{\text{Force} \times (\text{distance})^2}{(\text{Pole strength})^2}$	$\frac{[M^1L^1T^{-2}] \times [L^2]}{[L^1A^1]^2} = [M^1L^1T^{-2}A^{-2}]$	NA^{-2}

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CLASSIFICATION OF PHYSICAL QUANTITY →

On the basis of dimensional analysis, the physical quantities have been classified into four categories.

(a) Dimensional Variable →

These are the quantities which are liable to variations and posses dimensions.

Ex → Velocity, acceleration, force, work, etc.

(b) Dimensionless Variables →

These are the quantities which are liable to variations and do not posses dimensions.

Ex Angle, strain, specific gravity etc.

(c) Dimensional constant →

These are the quantities which do not change and posses dimensions.

Ex Gravitational Constant (G), Plancks constant (h) Gas constant (R), Boltzmann constant (K) etc.

(d) Dimensionless Constant →

These are the quantities which are Constants & do not posses any dimension.

Ex – Natural no (1, 2, 3, ...), π, e etc.

Application Of Dimensional Analysis →

There are 3 uses of Dimensional Analysis.

(a) To check the correctness of a given relation.

(b) To convert the values of a physical quantity from one system to another.

(c) To derive a relation between various physical quantities.

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...these based on a Principle ...

which can be stated as

Def^n → It states that the dimensional formula of every term on the two sides of a correct relation must be same.

1.5 (a) To check the correctness of a given relation

The dimensions of all terms on the LHS of equation are compared with these on the RHS. If the dimensions of the physical quantities on LHS are the same as those on the RHS the equation is correct or dimensionally homogeneous.

Ex-① Let us check the relation

$$S = ut + \frac{1}{2}at^2$$

Dimensional formula of $S = [L^1]$

$$ut = [LT^{-1}] \times [T^1] = [L^1]$$

$$\frac{1}{2}at^2 = [LT^{-2}][T^1]^2$$

$$= [L^1T^{-2}][T^2]$$

$$= [L^1]$$

Where $\frac{1}{2}$ → Dimensionless constant

Putting the dimensional formula for all terms

$$[L^1] = [L^1] + [L^1]$$

Hence dimension of L is same on both sides & [M] & [T] are absent on both sides.

(2) Let us check the relation

$$v = u + at$$

Dimensional formula of velocity $= [L^1T^{-1}]$

acceleration $= [L^1T^{-2}]$

Time $= [T^1]$

Putting the dimensional formula of all terms

$$[L^1T^{-1}] = [L^1T^{-1}] + [L^1T^{-2}][T^1]$$

$$= [L^1T^{-1}] + [L^1T^{-1}]$$

Hence dimension of both the terms L & T are same on both sides & [M] is absent on both sides.

(3) Let us check the relation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Dimensional formula of time period $= [T^1]$

Length $= [L^1]$

Acceleration due to gravity $= [L^1T^{-2}]$

$$[T^1] = \sqrt{\frac{[L^1]}{[L^1T^{-2}]}}$$

$$= \sqrt{[L^0T^2]}$$

$$= [T^1]$$

Hence dimension of the term T are same on both sides & [M] & [L] are absent on both sides.

(b) Convert the value of a Physical Quantity from one system to another

As the dimensions are same in both systems of units, the dimensional formula in both cases can be equated in following manner.

$$n_1[M_1^x L_1^y T_1^z] = n_2[M_2^x L_2^y T_2^z]$$

where n_1 & n_2 are pure nos which measure of the physical quantity in respective systems.

M_1, L_1 & T_1 are units in System-I

M_2, L_2 & T_2 are units in System II

from above systems

$$\left(\frac{n_2}{n_1}\right) = \left[\frac{M_1}{M_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T_1}{T_2}\right]^z$$

$$\Rightarrow \boxed{n_2 = n_1\left(\left[\frac{M_1}{M_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T_1}{T_2}\right]^z\right)} \quad \text{--- (1)}$$

Ex-1

Let us convert a force of 1 Newton (SI System) to dyne (CGS system)

The dimensional formula of force is $[M^1L^1T^{-2}]$

System-I (MKS)	FORCE $[M^1L^1T^{-2}]$	SYSTEM-2 (CGS)
$M_1 = 1kg$	$x = 1$	$M_2 = 1gm$
$L_1 = 1m$	$y = 1$	$L_2 = 1cm$
$T_1 = 1s$	$z = -2$	$T_2 = 1s$
$n_1 = 1$		$n_2 = ?$

Substituting the values in equn-(1)

$$n_2 = 1\left(\left[\frac{1kg}{1gm}\right]^1 \left[\frac{1m}{1cm}\right]^1 \left[\frac{1s}{1s}\right]^{-2}\right)$$

$$= \left(\left[\frac{1000gm}{1gm}\right]\left[\frac{100cm}{1cm}\right]\left[\frac{1s}{1s}\right]^{-2}\right)$$

$$= 10^3 \times 10^2 \frac{gm\ cm}{sec^2}$$

$$\Rightarrow n_2 = 10^5 \text{ dyne}$$

$\therefore$ $\boxed{1 \text{ Newton} = 10^5 \text{ dyne}}$

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Ex-2 Let us convert a work of 1 Joule into erg (13)

The dimensional formula of work is $[M^1L^2T^{-2}]$

SYSTEM-1 (MKS)	Work $[M^1L^2T^{-2}]$	SYSTEM-2 (CGS)
$M_1 = 1kg$	$x = 1$	$M_2 = 1gm$
$L_1 = 1m$	$y = 2$	$L_2 = 1cm$
$T_1 = 1s$	$z = -2$	$T_2 = 1s$
$n_1 = 1$		$n_2 = ?$

Substituting these values in eqn (1), we get

$$n_2 = 1\left(\left[\frac{1kg}{1gm}\right]^1\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}\right)$$

$$= \left(\left[\frac{1000gm}{1gm}\right]\left[\frac{100cm}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}\right)$$

$$= 10^3 \times (10^2)^2 \frac{gm\,cm^2}{sec^2}$$

$$= 10^3 \times 10^4 \frac{gm\,cm^2}{sec^2}$$

$$= 10^7 \text{ Erg}$$

$$\therefore \boxed{n_2 = 10^7 \text{ Erg}}$$

$$\therefore \boxed{1 \text{ Joule} = 10^7 \text{ Erg}}$$

(c) Derive the formula

(1) Let us an obtain an expression for Centripetal force required to move to body of mass m, with velocity v in a circle of radius r.

Let the centripetal force 'f' depends on m, v & r.

$\therefore F \propto m^a$
$\propto v^b$
$\propto r^c$

where, a, b, c are the dimensionless constants

combining we get,

$F \propto m^a . v^b . r^c$

$\Rightarrow \boxed{F = K m^a v^b r^c}$ — (1)

putting the dimensional formula of the quantities, we get

$[M^1L^1T^{-2}] = [M^0L^0T^0][M^1]^a[L^1T^{-1}]^b[L^1]^c$

$\Rightarrow [M^1L^1T^{-2}] = [M^a L^{b+c} T^{-b}]$

Equating the dimensions of M, L & T on both sides

$\boxed{a = 1}$

$b + c = 1$ — (2)

$-b = -2 \Rightarrow \boxed{b = 2}$

Substituting the value of b in eqn (2), we get

$2 + c = 1$

$\Rightarrow \boxed{c = -1}$

putting the value of a, b & c in eqn (1) we get

$F = K m^1 . v^2 . r^{-1}$

$\Rightarrow F = K \frac{mv^2}{r}$

Taking $K = 1$, the eqn becomes $\boxed{F = \frac{mv^2}{r}}$

This is the required expression for centripetal force

2) Wavelength of a particle (λ) depends upon planck's constant (h), mass (m) & velocity (v);

$\therefore \lambda \propto h^a$ — (1)

$\propto m^b$ — (2)

$\propto v^c$ — (3)

Combining we get,

$$\lambda \propto h^a \cdot m^b \cdot v^c$$

$$\Rightarrow \boxed{\lambda = K h^a m^b \cdot v^c} - (4)$$

putting the dimensional formula on both sides,

$$[L^1] = [M^0 L^0 T^0][M^1 L^2 T^{-1}]^a [M^1]^b [L^1 T^{-1}]^c$$

$$\Rightarrow [M^0 L^1 T^0] = [M^a L^{2a} T^{-a}][M^b][L^c T^{-c}]$$

$$\Rightarrow [M^0 L^1 T^0] = [M^{a+b} L^{2a+c} T^{-a-c}]$$

Equating the dimensions of M, L & T in both sides,

$$a+b=0 - (5)$$
$$2a+c=1 - (6)$$
$$-a-c=0 - (7)$$

From equn - (6) & (7)

$$\boxed{a=1}$$

putting the value of 'a' in equn - (5)

$$1+b=0$$
$$\Rightarrow \boxed{b=-1}$$

putting the value of 'a' in equn (6)

$$2\times 1+c=1$$
$$\Rightarrow 2+c=1$$
$$\Rightarrow \boxed{c=-1}$$

Putting the value of a, b & c in equn (4)

$$\lambda = K h^1 \cdot m^{-1} \cdot v^{-1}$$

$$\Rightarrow \lambda = K \frac{h}{mv}$$

Take $K=1$, the equn becomes $\boxed{\lambda = \frac{h}{mv}}$

This is the required expression for wave length.

3> Time period of a simple pendulum depends upon length of a bob(l) & acceleration due to gravity (g). (16)

$T \propto l^a$ — (1)

$\propto g^b$ — (2)

$\therefore T \propto l^a \cdot g^b$

$\Rightarrow \boxed{T = K l^a \cdot g^b}$ — (3)

putting the dimensional formula on both sides

$[T^1] = [M^0L^0T^0][L^1]^a[L^1T^{-2}]^b$

$\Rightarrow [M^0L^0T^1] = [M^0L^0T^0][L^a][L^bT^{-2b}]$

$\Rightarrow [M^0L^0T^1] = [M^0L^{a+b}T^{-2b}]$

Equating the dimensional formula of M, L & T we get

$a+b = 0$ — (4)

$-2b = 1$ — (5) $\Rightarrow \boxed{b = -\frac{1}{2}}$

putting the value of b in eqn (4)

$a - \frac{1}{2} = 0$

$\Rightarrow \boxed{a = \frac{1}{2}}$

Substituting the value of a & b in eqn (3)

$T = K l^{1/2} \cdot g^{-1/2}$

$\Rightarrow T = K \frac{l^{1/2}}{g^{1/2}}$

$\Rightarrow \boxed{T = K\sqrt{\frac{l}{g}}}$

Take $K = 2\pi$, the eqn becomes $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

This is the required expression for Time period.

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LIMITATIONS OF DIMENSIONAL ANALYSIS →

(1) It gives no information wheather a physical quantity is a scaler or vector.

(2) It gives no information about dimensionless constants like 1, 2, 3 ... π, e.

(3) It gives no information about dimensional constants like G, h, K etc.

(4) This method is applicable only for power functions but it fails to derive relations of quantities involving exponential & trignometric function.

(5) This method fails to derive formula of a physical quantity which depends upon more than three factors.

(6) The method of dimension can not be derived the exact form of relation.

(7) The method can not be directly applied to derive relations which contain more than one term on one side or both sides of the equations, such as $v = u + at$ or $S = ut + \frac{1}{2}at^2$ etc. such relations can be derived indirectly.

(8) A dimensionally correct relation may not be true physical relation because the dimensionally equality is not sufficient for the correctness of a given relation.

UNIT- 2
SCALARS & VECTORS

2.1 Different physical quantities can be classified into two categories

(i) Scalar Quantities →

Scalar quantities are those quantities which require only the magnitude for their complete specifications.

Ex – Mass, length, volume, density, temp, energy, electric charge etc.

(ii) Vector quantities →

Vector quantities are those quantities which require magnitude as well as direction for their complete specification.

A directed line segment is called Vector.

Ex → Displacement, velocity, Acceleration, force etc.

REPRESENTATION OF A VECTOR →

→ vectors are represented by a straight line with arrow head on it i.e an arrowed line.

→ starting point of an arrowed line is called tail or origin of vector, and the end of the arrowed line is called top or head of the vector.

→ vector can be represented by a single letter with arrow head on it.

Ex

Force – $\vec{F}$

velocity – $\vec{v}$

Acceleration – $\vec{a}$

Work – $\vec{W}$

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TYPES OF VECTORS →

(a) Zero vector or Null vector →

→ All vectors have initial & terminal points.

→ A zero vector or a null vector is one in which these two points coincide.

→ It is denoted by $\vec{0}$.

→ Since the magnitude is zero, we can not assign a direction to these vectors.

Def^n → It is a vector having zero magnitude and an arbitrary direction.

Ex – $\vec{AA}$, $\vec{BB}$ etc.

(b) Unit Vector →

→ It is a vector having a magnitude of unity or 1 unit.

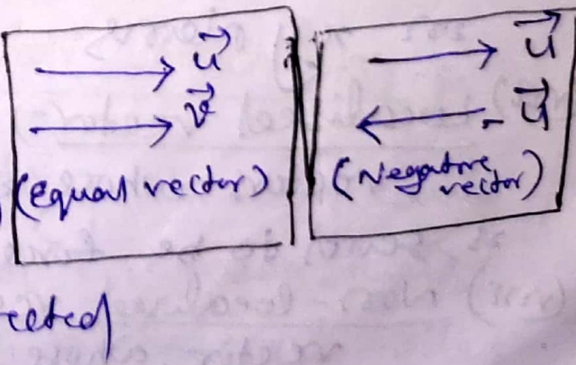
→ A unit vector in the direction of a given vector $\vec{a}$ is denoted as $\hat{a}$.

(c) Equal vector →

If two vectors $\vec{a}$ & $\vec{b}$ have the same magnitude and direction regardless the positions of their initial points, then they are equal vectors.

These vectors are written as $\vec{a} = \vec{b}$

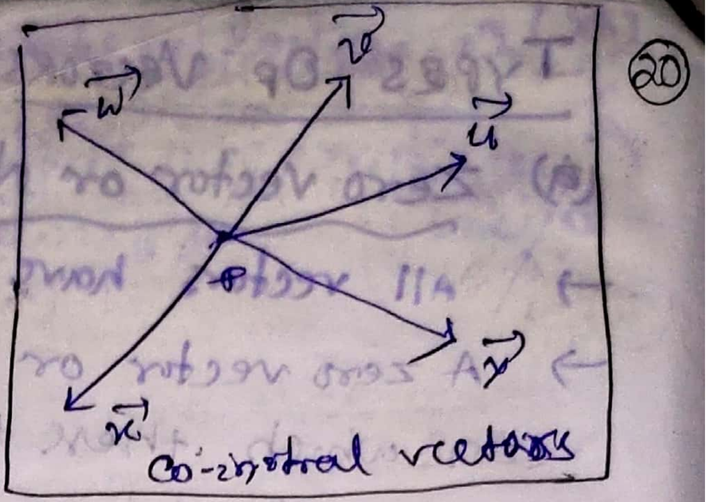
(d) Negative vector →



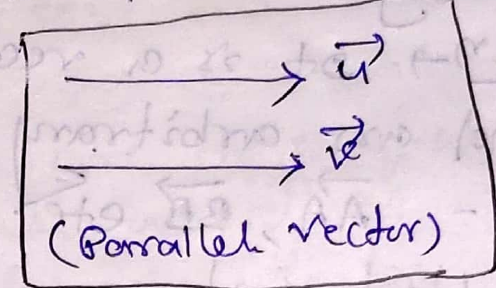
A vector is said to be a negative vector if it is represented by a line having same length as that of the second & is directed in opposite direction.

Ex $\vec{BA} = -\vec{AB}$

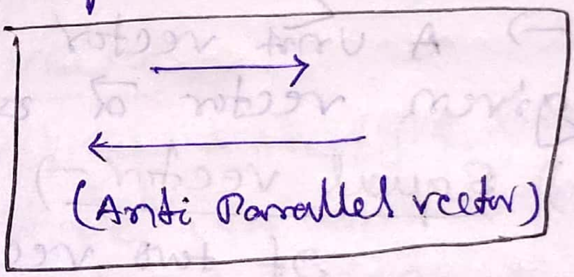
(v) Collinear Vectors →
vectors having a common line of action are called collinear vectors.
There are two types of collinear vectors.



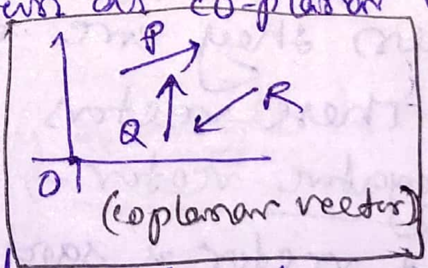
(a) Parallel vectors →
Two vectors (which may have different magnitudes) acting along same direction are called parallel vectors.
→ vector $\vec{u}$ & $\vec{v}$ are parallel vectors. Angle between them is zero



(b) Anti-Parallel vectors →
Two vectors which are directed in opposite directions are called antiparallel vectors.
→ vectors $\vec{u}$ & $\vec{v}$ are called antiparallel vectors.
Angle between them is 180.



(vi) Co-Planar Vectors →
vectors situated in one plane irrespective of their direction are known as co-planar vector, vectors $\vec{P}$, $\vec{Q}$ & $\vec{R}$ drawn in x-y plane



(vii) Localised vector →
vectors whose initial point is fixed is said to be fixed vector or localised vector.
(viii) Non-localised vector →
vector whose initial point is not fixed is said to be non-localised vector or a free vector
en - ... momentum, impulse.

(ix) Co-initial vector →

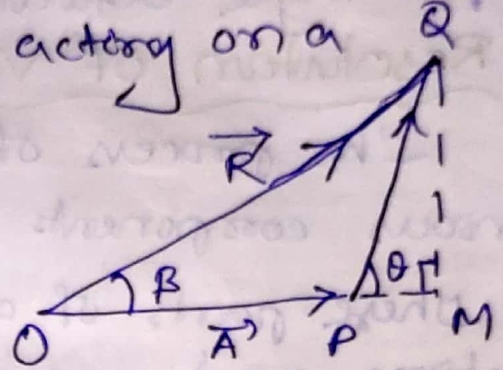
→ A number of vectors having a common initial point are called co-initial vector.

→ vectors $\vec{u}, \vec{v}, \vec{w}, \vec{x}$ & $\vec{y}$ all meeting at a point P are co-initial vector.

2.2 TRIANGLE LAW OF VECTOR ADDITION →

Statement →

"If two vectors are acting on a particle at the same time are represented in magnitude & direction by the two sides of a triangle taken in same order, then their resultant is represented in magnitude & direction by the third side of the triangle taken in opposite order."



→ This is a law for addition of two vectors.

$$R = \sqrt{A^2+B^2+2AB\cos\theta} \quad \text{--- (1)}$$

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta} \quad \text{--- (2)}$$

Equation (1) & (2) give the magnitude & direction respectively of the Resultant R.

Parallelogram law of Vector Addition →

" If two vectors are acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point,

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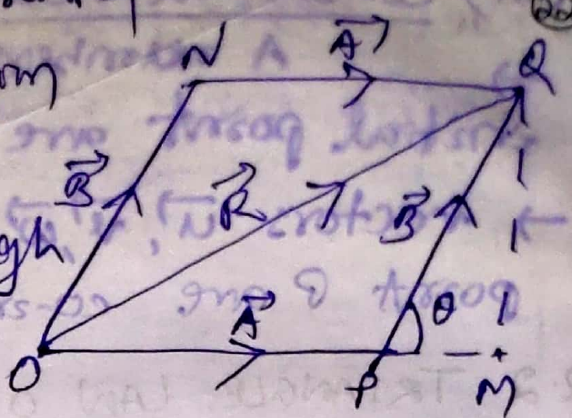
then Resultant is represented in magnitude and direction by the diagonal of a parallelogram passing through that point".



θ → Angle between $\vec{A}$ & $\vec{B}$

β → Angle between $\vec{R}$ & $\vec{A}$

2.3 Resolution of Vector →

The process of splitting a vector into various components is called Resolution of vector

These parts of a vector may act in different directions and are called "Components of vector".

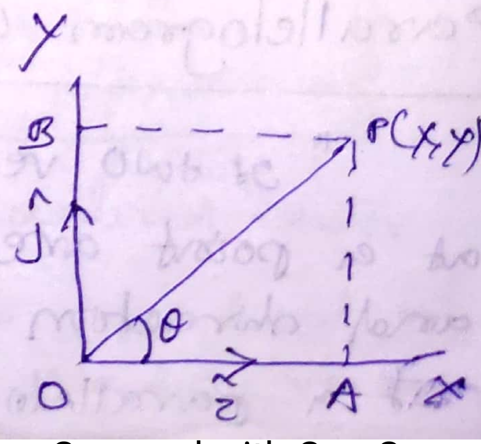
A vector can be resolved into a no. of Component. Generally there are three components of vector.

Component along x-axis called x-component
Component along y-axis called y-component
Component along z-axis called z-component

Let us consider only two components x-component and y-component which are perpendicular to each other. These components are called Rectangular components of vector.

RECTANGULAR COMPONENTS →

→ Rectangular components of a given vector are its components in two mutually perpendicular directions in the plane of the given vectors.



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Let $\vec{OP}(=\vec{R})$ be position vector of a point $P(x,y)$. From 'P' draw PA & PB ⊥rs on x-axis & y-axis respectively.

Thus $OA = x$ & $OB = y$

If $\hat{i}$ & $\hat{j}$ are the unit vectors along x-axis & y-axis.

$$\vec{OA} = x\hat{i} \text{ \& } \vec{OB} = y\hat{j}$$

Applying Triangle's law in triangle OAP

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\Rightarrow \vec{OP} = \vec{OA} + \vec{OB}$$

$$\Rightarrow \vec{R} = x\hat{i} + y\hat{j}$$

$\vec{OA}\,(x\hat{i})$ & $\vec{OB}\,(y\hat{j})$ are called x-component & y-component of $\vec{R}$ respectively.

Let θ be the angle which the given vector $\vec{R}$ makes with x-axis

In $\triangle OAP$,

$$\frac{OA}{OP} = \cos\theta \qquad \frac{AP}{OP} = \sin\theta$$

$$\text{or } \frac{OB}{OP} = \sin\theta$$

$$\Rightarrow \frac{x}{R} = \cos\theta \qquad \Rightarrow \frac{y}{R} = \sin\theta$$

$$\Rightarrow \boxed{x = R\cos\theta} \qquad \Rightarrow \boxed{y = R\sin\theta}$$

$$\therefore \vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

2.4 PRODUCT OF TWO VECTORS →

Two vectors $\vec{A}$ & $\vec{B}$ inclined to each other that an angle θ can be multiplied in two ways.

(i) Dot Product or Scalar product →

It is given by the product of their magnitude and cosine of angle between the two

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$

It is a scalar quantity

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Properties of Dot Product –

i) It obeys commutative law

i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

ii) It obeys distributive law

i.e. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

iii) If two vectors are $\perp r$, their dot product is zero.

i.e. $\boxed{\vec{A}\cdot\vec{B} = 0}$

iv) If two vectors are parallel, their dot product is maximum.

i.e. $\boxed{\vec{A}\cdot\vec{B} = AB\cos 0 = AB}$

v) If two vectors have the same magnitude & direction then $\vec{A}\cdot\vec{A} = AA\cos 0 = A^2$

vi) If the angle between two vectors is acute, their dot product is +ve.

vii) If the angle between two vectors is obtuse, their dot product is -ve.

viii) If $\hat{i}, \hat{j}$ & $\hat{k}$ are unit vectors, then

$$\boxed{\begin{array}{l}\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \\ \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0\end{array}}$$

ix) Dot product does not obey Associative law.

x) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)$

xi) If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

xii) $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$|A| = A = \sqrt{A_1^2 + A_2^2 + A_3^2}$

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iii) Cross product or vector product →

It is defined as the product of their individual magnitude and the sine of angle between them. Its direction is always ⊥r to the plane containing two vectors.

$$\boxed{\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}}$$

where $\hat{n}$ → Unit vector along the normal containing two vectors.

It is a vector quantity.

Properties of Cross Product →

i) It does not obey commutative law.
   i.e $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ but $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

ii) The cross product obey distributive law
   i.e $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

iii) If $\vec{A}$ & $\vec{B}$ are collinear (i.e $\theta = 0°$ & $180°$) then their vector product yields null vector.

$$\vec{A}\times\vec{B} = 0$$

iv) If two vectors are ⊥r to each other, then

$$\vec{A}\times\vec{B} = AB\sin 90 = AB$$

v) The cross product of a vector with itself a null vector.
   i.e $\boxed{\vec{A}\times\vec{A} = 0}$

vi) If $\hat{i}$, $\hat{j}$ & $\hat{k}$ are unit vectors along the axes, then

$$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$$

vii) $\hat{i}\times\hat{j} = \hat{k} \Rightarrow \hat{j}\times\hat{i} = -\hat{k}$
   $\hat{j}\times\hat{k} = \hat{i} \Rightarrow \hat{k}\times\hat{j} = -\hat{i}$
   $\hat{k}\times\hat{i} = \hat{j} \Rightarrow \hat{i}\times\hat{k} = -\hat{j}$

viii) $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$$|\vec{A}\times\vec{B}| = AB\sin\theta$$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

ix) If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$$|\vec{A}\times\vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \hat{i}(A_2B_3 - A_3B_2) - \hat{j}(A_1B_3 - A_3B_1) + \hat{k}(A_1B_2 - A_2B_1)$$

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SIMPLE NUMERICAL PROBLEMS

1) Q) Two vectors 5N & 20N are acting at an angle of 120° between them. Find the resultant force in magnitude & direction.

Soln We know, $\vec{R} = \sqrt{A^2+B^2+2AB\cos\theta}$

Given $F_1 = 5N$, $F_2 = 20N$, $\theta = 120°$

$\therefore R = \sqrt{(5)^2+(20)^2+2\times5\times20\times\cos 120}$

$= \sqrt{25+400+2\times100\times(-\frac{1}{2})}$

$= \sqrt{425-100}$

$= \sqrt{325}$

$\Rightarrow \boxed{R = 18.03N}$

$\tan\beta = \frac{F_2\sin\theta}{F_1+F_2\cos\theta}$

$= \frac{20\sin 120}{5+20\cos 120°}$

$= \frac{20\times\frac{\sqrt{3}}{2}}{5+20(-\frac{1}{2})}$

$= \frac{10\sqrt{3}}{-5}$

$\Rightarrow \tan\beta = -2\sqrt{3}$

$\Rightarrow \beta = \tan^{-1}(-2\sqrt{3})$

$\Rightarrow \boxed{\beta = 106°6'}$ with 5N force

2) Q) Two forces equal in magnitude, have magnitude of their resultant equal to either. Find the angle between them.

Soln Here $F_1 = F_2 = R = F$

We know, $R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$\Rightarrow F_1 = \sqrt{F_1^2+F_1^2+2F_1F_1\cos\theta}$

$\Rightarrow F_1 = \sqrt{2F_1^2+2F_1^2\cos\theta}$

$\Rightarrow F_1 = \sqrt{2F_1^2(1+\cos\theta)}$

$\Rightarrow F_1^2 = 2F_1^2(1+\cos\theta)$

$\Rightarrow \frac{1}{2} = 1+\cos\theta$

$\Rightarrow \frac{1}{2}-1 = \cos\theta$

$\Rightarrow -\frac{1}{2} = \cos\theta$

or $\cos\theta = -\frac{1}{2} = \cos 120°$

$\Rightarrow \boxed{\theta = 120°}$

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3) Q) At what angle two force $(P+Q)$ & $(P-Q)$ should be inclined so as to have resultant $\sqrt{3P^2+Q^2}$.

Soln $F_1 = P+Q$, $F_2 = P-Q$, $R = \sqrt{3P^2+Q^2}$

$R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$\Rightarrow \sqrt{3P^2+Q^2} = \sqrt{(P+Q)^2+(P-Q)^2+2(P+Q)(P-Q)\cos\theta}$

$\Rightarrow \sqrt{3P^2+Q^2} = \sqrt{P^2+Q^2+\cancel{2PQ}+P^2+Q^2-\cancel{2PQ}+2(P^2-Q^2)\cos\theta}$

$\Rightarrow \sqrt{3P^2+Q^2} = \sqrt{2P^2+2Q^2+2(P^2-Q^2)\cos\theta}$

Squaring on both sides

$3P^2+Q^2 = 2P^2+2Q^2+2(P^2-Q^2)\cos\theta$

$\Rightarrow 3P^2-2P^2+Q^2-2Q^2 = 2(P^2-Q^2)\cos\theta$

$\Rightarrow \cancel{(P^2-Q^2)} = 2\cancel{(P^2-Q^2)}\cos\theta$

$\Rightarrow 1 = 2\cos\theta$

$\Rightarrow \frac{1}{2} = \cos\theta$

or $\cos\theta = \frac{1}{2} = \cos 60°$

$\boxed{\theta = 60°}$

4) Q) Resultant of two forces equal in magnitude at right angles to each other is 1414N. Find the magnitude of each force.

Soln $R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

Given $F_1 = F_2$, $R = 1414N$, $\theta = 90°$

$\Rightarrow 1414 = \sqrt{F_1^2+F_1^2+2F_1F_1\cos 90}$

$\Rightarrow 1414 = \sqrt{2F_1^2+2F_1^2\times 0}$

$\Rightarrow 1414 = \sqrt{2F_1^2}$

$\Rightarrow 1414 = \sqrt{2}F_1$

$\Rightarrow F_1 = \frac{1414}{\sqrt{2}}$

$\Rightarrow \boxed{F_1 = 999.84N}$

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5) Two forces whose magnitude are in the ratio 3 : 5 give a resultant of 28N. If the angle of their inclination is 60°. Find the magnitude of each force.

Soln

$F_1 = 3x$, $F_2 = 5x$, $R = 28N$, $\theta = 60^\circ$

We know, $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{(3x)^2 + (5x)^2 + 2 \cdot 3x \cdot 5x \cdot \cos 60^\circ}$

$= \sqrt{9x^2 + 25x^2 + 2 \times 15x^2 \times \frac{1}{2}}$

$= \sqrt{34x^2 + 15x^2}$

$= \sqrt{49x^2}$

$\Rightarrow 28 = 7x$

$\Rightarrow x = 4$

$\therefore F_1 = 3x = 3 \times 4 = 12N$

$F_2 = 5x = 5 \times 4 = 20N$

6) Q) The greatest & lowest resultant of the two forces acting at a point is 10N & 6N respectively. If each force is increased by 3N. Find the resultant of two forces when acting at a point at an angle of 90° with each other.

Soln

$R = F_1 + F_2 = 10N$

$R = F_1 - F_2 = 6N$

$2F_1 = 16N \Rightarrow F_1 = 8N$

$\therefore F_2 = 2N$

$F_1' = 8N + 3N = 11N$

$F_2' = 2N + 3N = 5N$

$\theta = 90^\circ$

$\therefore R = \sqrt{11^2 + 5^2 + 2 \times 11 \times 5 \times \cos 90^\circ}$

$= \sqrt{121 + 25}$

$= \sqrt{146}$

$\Rightarrow \boxed{R = 12.08N}$

$\tan\beta = \frac{F_2' \sin\theta}{F_1' + F_2' \cos\theta}$

$= \frac{5\sin 90}{11 + 5\cos 90}$

$= \frac{5 \times 1}{11}$

$\tan\beta = \frac{5}{11} \Rightarrow \beta = \tan^{-1}\left(\frac{5}{11}\right)$

$\Rightarrow \boxed{\beta = 24.44^\circ}$

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(29)

7) Q) Find the rectangular components of a velocity 8 m/s when one of the components makes an angle of 30° with the resultant.

Soln

$x = v\cos\theta$

$y = v\sin\theta$

Given $v = 8$ m/s, $\theta = 30°$

$x = 8\cos 30° \Rightarrow x = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$ m/s

$y = 8\sin 30° \Rightarrow y = 8 \times \frac{1}{2} = 4$ m/s

8) Q) One of the rectangular components of a force of 65 N is 25 N. Find the other component.

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Soln

$x = F\cos\theta$

$y = F\sin\theta$

Given $x = 25$ N, $F = 65$ N

$\Rightarrow 25 = 65\cos\theta$

$\Rightarrow \frac{25}{65} = \cos\theta$

or $\boxed{\cos\theta = \frac{25}{65}}$ or $\boxed{\cos\theta = \frac{5}{13}}$

We know $\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1 - \frac{25^2}{65^2}}$

$= \sqrt{1 - \frac{625}{4225}}$

$= \sqrt{\frac{4225-625}{4225}}$

$= \sqrt{\frac{3600}{4225}}$

$= \sqrt{\frac{60^2}{65^2}}$

$\boxed{\sin\theta = \frac{60}{65}}$

$\therefore y = \frac{\cancel{65} \times 60}{\cancel{65}} = 60$

$\Rightarrow \boxed{y = 60 N}$

$\therefore$ other component is 60 N

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1 - \frac{25}{169}}$

$= \sqrt{\frac{169-25}{169}}$

$= \sqrt{\frac{144}{169}}$

$= \sqrt{\frac{12^2}{13^2}}$

$\boxed{\sin\theta = \frac{12}{13}}$

$\therefore y = 65 \times \frac{12}{13}$

$\boxed{y = 60 N}$

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9) Q) At what angle dot product of two vector must be equal to the Cross product of two vectors. (30)

Soln

$\vec{A}\cdot\vec{B} = \vec{A}\times\vec{B}$

$\Rightarrow AB\cos\theta = AB\sin\theta$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45^\circ$

$\boxed{\theta = 45^\circ}$

10) Q) Given $\vec{A} = 5\hat{i}+7\hat{j}$, $\vec{B} = 2\hat{i}+9\hat{j}$ [2013(W)-NEW]

Find $\vec{A}\times\vec{B}$ & $\vec{A}\cdot\vec{B}$

Soln

$\vec{A}\times\vec{B}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 7 & 0 \\ 2 & 9 & 0 \end{vmatrix}$$

$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(45-14)$

$= 0\hat{i} - 0\hat{j} + 31\hat{k}$

$= 31\hat{k}$

$\vec{A}\cdot\vec{B}$

$= (5\hat{i}+7\hat{j})\cdot(2\hat{i}+9\hat{j})$

$= 10 + 63$

$= 73$ Unit

11) Q) Given $\vec{A} = 2\hat{i}+2\hat{j}+5\hat{k}$, $\vec{B} = 3\hat{i}-7\hat{j}$ [2018(S) New 70]

Find $\vec{A}\times\vec{B}$

Soln

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -7 & 0 \end{vmatrix}$$

$= \hat{i}(0-(-35)) - \hat{j}(0-15) + \hat{k}(-14-9)$

$= 35\hat{i} + 15\hat{j} - 23\hat{k}$

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KINEMATICS

3.1 Concept of Rest and Motion

· Particle →

A particle can be defined as a sphere of zero radius,

Rest & Motion →

A body is said to be at rest if it does not change its position with respect to its surroundings.

A body is said to be at motion if it change its position with respect to its surroundings

3.2 Displacement →

Displacement of a body is a vector connecting the initial and final positions of the body & is directed away from initial towards the final position.

Speed →

Speed of a body is defined as the distance covered by the body in 1sec.

If Δs is the distance travelled by a body in a time Δt.

$$\text{Average Speed} = \frac{\Delta s}{\Delta t}$$

If the time interval Δt is chosen to be very small i.e. $\Delta t \to 0$, the corresponding Speed is called Instantaneous Speed

$$\text{Inst. Speed} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

It is a scalar quantity

Unit MKS – m/s

CGS – cm/s

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Velocity →

Velocity of a body is defined as the rate of change of displacement.

$$\text{velocity } \vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

If the time interval Δt is very small, the corresponding value of velocity is called instantaneous velocity 'v'.

$$v = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Unit MKS – m/s
CGS – cm/s

a) **Uniform velocity →**

Velocity of a body is said to be uniform if it covers equal displacement in equal interval of time.

$$\vec{v}_{av} = \frac{\Delta x}{\Delta t}$$

$$\text{or } \vec{v} = \frac{d\vec{x}}{dt}$$

b) **Non-Uniform Velocity →**

Velocity of a body is said to be non-uniform if it covers unequal displacement in equal interval of time.

Acceleration →

Acceleration is defined as the change in velocity in one second.

Or

Acceleration is defined as the rate of change of velocity.

a) **Uniform Acceleration →**

The acceleration of a body is said to be uniform if its velocity changes by equal amounts of in equal interval.

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

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5) Non-Uniform Acceleration →

The acceleration of a body is said to be non-uniform if its velocity changes by unequal amounts in equal intervals of time.

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

FORCE →

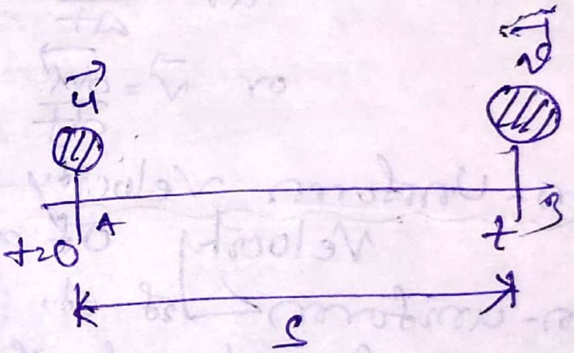
In physics, a force is any interaction that, when unopposed, will change the motion of an object. A force can cause an object with mass to change its velocity i.e to accelerate. A force has both magnitude & direction, making it a vector quantity $\boxed{F = m a}$

SI $kg\, m/s^2$

CGS Newton(N)

Equation of Motion of a straight line →

Consider a body moving past a point with velocity $\vec{u}$ at $t=0$. It is called initial velocity. It reaches a point B after t second and thus acquires a velocity $\vec{v}$ due to the uniform acceleration $\vec{a}$ present on the body, $\vec{v}$ is called the final velocity. It covers distance 's' in going from A to B.



Equations involving $\vec{u}, \vec{v}, \vec{a}, \vec{s}$ & t are called Equation of motion.

a) velocity after a time t is $\boxed{\vec{v} = \vec{u} + \vec{a}t}$

b) Distance travelled in t second $\boxed{\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2}$

c) velocity after travelling a distance 's' is $\boxed{v^2 - u^2 = 2as}$

d) Displacement in nth second, $\boxed{\vec{S}_{nth} = \vec{u} + \frac{\vec{a}}{2}(2n-1)}$

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3.3 EQUATION OF MOTION UNDER GRAVITY →

Downward motion →

i) $v = u + gt$

ii) $h = ut + \frac{1}{2}gt^2$

iii) $v^2 - u^2 = 2gh$

Upward Motion →

i) $v = u - gt$

ii) $h = ut - \frac{1}{2}gt^2$

iii) $v^2 - u^2 = -2gh$

3.4 CIRCULAR MOTION →

A body is said to move in circular motion if it moves in such a way that its distance from a fixed point always remains constant.

UNIFORM CIRCULAR MOTION :→

A body is said to move in uniform circular motion if it moves in a circular motion with uniform velocity.

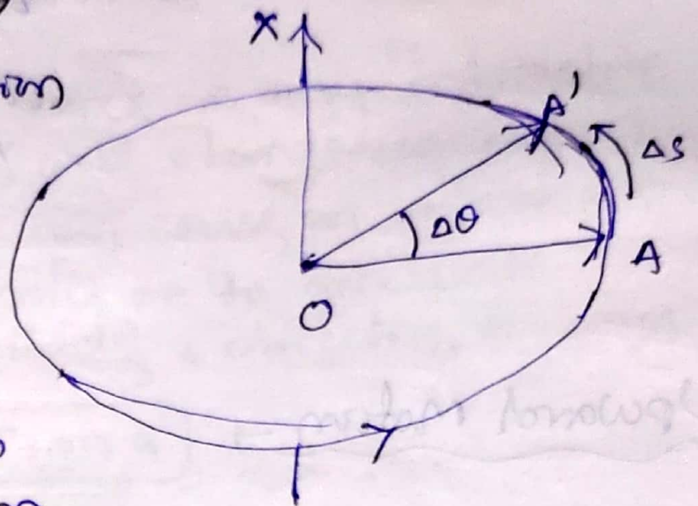
ROTATIONAL MOTION :→

A body is said to move in rotational if all the particles of the body move in circles, the centers of which are situated on a straight line called Axis of Rotation

PATH OF THE PARTICLE →

The path of a particle undergoing rotational motion is a circle. All the particles of a body move in concentric circles.

AXIS OF ROTATION →



Axis of rotation of a particle is a straight line passing through the centre of the circle in which the particle rotates and is ⊥r to the plane of rotation.

ANGULAR DISPLACEMENT → ($\Delta\vec{\theta}$)

Angular Displacement of a particle undergoing rotational motion is defined as the angle turned by its radius vector.

$$\therefore \Delta\theta = \frac{\Delta S}{r}$$

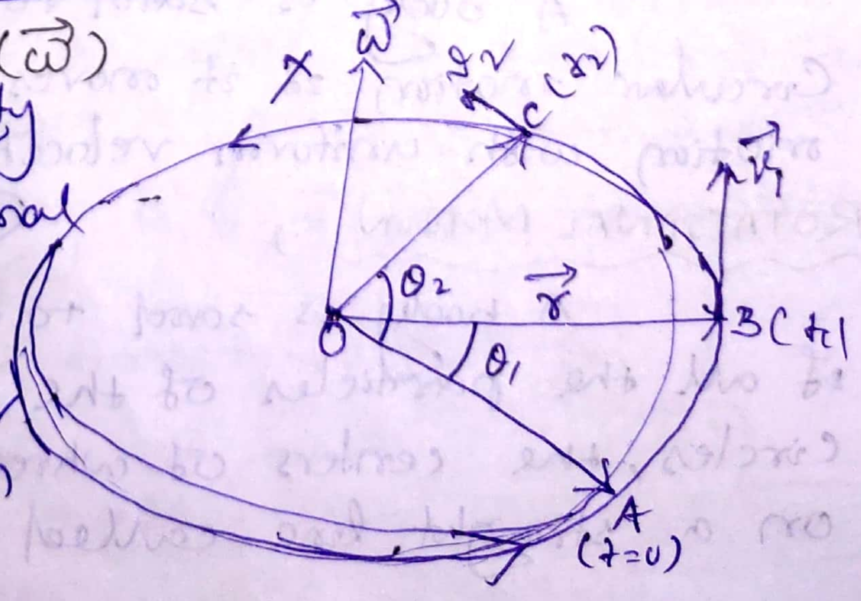
$$\Rightarrow \boxed{\Delta S = r\Delta\theta}$$

where $\Delta S \rightarrow$ Linear Displacement of particle

vector form $\boxed{\vec{\Delta S} = \vec{\Delta\theta} \times \vec{r}}$

ANGULAR VELOCITY ($\vec{\omega}$)

Angular velocity of a particle undergoing rotational motion is defined as the rate of change of angular displacement with time.



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Let the particle moves from A to B in a time t_1 & to C in time t_2.

If $\vec{\theta}_1$ & $\vec{\theta}_2$ are its angular displacements at these instants, its average angular velocity 'ω_{av}' is

$$'\omega_{av}' = \frac{\vec{\theta}_2 - \vec{\theta}_1}{t_2 - t_1} = \frac{\vec{\Delta\theta}}{\Delta t}$$

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\vec{\Delta\theta}}{\Delta t} = \frac{\vec{d\theta}}{dt}$$

$$\therefore \boxed{\vec{\omega} = \frac{\vec{d\theta}}{dt}}$$

ANGULAR ACCELERATION ($\vec{\alpha}$):→

Angular acceleration of a body is defined as the rate of change of its angular velocity with time.

If $\vec{\omega}_1$ & $\vec{\omega}_2$ are the angular velocities of the particle A & B, average angular acceleration $\vec{\alpha}_{av}$ is

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$$

$$\vec{\alpha}_{av} = \frac{\vec{\Delta\omega}}{\Delta t}$$

If Δt is small ($\Delta t \to 0$), instantaneous acceleration $\vec{\alpha}$ is

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\vec{\Delta\omega}}{\Delta t} = \frac{\vec{d\omega}}{dt}$$

$$\boxed{\vec{\alpha} = \frac{\vec{d\omega}}{dt}}$$

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i) RELATION BETWEEN LINEAR VELOCITY (v) & ANGULAR VELOCITY (ω)

Scalar form

We know $AB = vt$ —①

Also $\theta = \frac{AB}{r}$

$\Rightarrow AB = r\theta$ —②

From eqn. ① & ②

$$vt = r\theta$$

$$\Rightarrow v = r\left(\frac{\theta}{t}\right)$$

$$\Rightarrow \boxed{v = r\omega} \quad \left(\because \omega = \frac{\theta}{t}\right)$$

Vector form $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$

ii) RELATION BETWEEN LINEAR ACCELERATION (a) & ANGULAR ACCELERATION (ω)

Angular acceleration of a body is defined as the rate of change of its angular velocity with time.

If $\vec{\omega}_1$ & $\vec{\omega}_2$ are the angular velocities of the particle A & B, therefore average angular acceleration $\vec{\alpha}_{av}$ is

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{\Delta t}$$

Scalar form

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

But $\omega_1 = \frac{v_1}{r}$ & $\omega_2 = \frac{v_2}{r}$

$$\therefore \alpha = \frac{\frac{v_2}{r} - \frac{v_1}{r}}{t_2 - t_1} = \frac{\frac{1}{r}(v_2 - v_1)}{t_2 - t_1}$$

$$\Rightarrow \alpha = \frac{1}{r}\left(\frac{v_2 - v_1}{t_2 - t_1}\right)$$

$$\Rightarrow \boxed{\alpha = \frac{a}{r}}$$

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Vector form

We know $\vec{v} = \vec{\omega} \times \vec{r}$

differentiating both sides we get, then

$$\frac{dv}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$$

$$\Rightarrow a = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

$$\Rightarrow \boxed{a = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}}$$

(i) Tangential Component $\rightarrow a_T = \vec{\alpha} \times \vec{r}$ (along tangent)

(ii) Radial Component $\rightarrow a_r = \vec{\omega} \times \vec{v}$ (along radius)

SIMPLE NUMERICAL PROBLEMS $\rightarrow$

1) Q) A body starts from rest and acquires a velocity 12 m/s in 5 second. Calculate the acceleration and the distance travelled.

Soln Initial velocity, $u = 0$

Final velocity, $v = 12$ m/s

Time taken, $t = 5$ sec

$\therefore$ Acceleration, $a = \frac{v-u}{t} = \frac{12-0}{5} = \frac{12}{5} = 2.4$ m/s^2

Since $s = ut + \frac{1}{2}at^2$

$= 0 \times 5 + \frac{1}{2} \times 2.4 \times 5^2$

$= 0 + 1.2 \times 25$

$$\boxed{s = 30.0 \text{ mt}}$$

2) Q) The velocity of a body increases at a constant rate, from 10 m/s to 25 m/s in 6 minute. Find the acceleration and the distance travelled.

Soln Initial velocity, $u = 10$ m/s (40)

Final velocity, $v = 25$ m/s

Time $= 6$ minute $= 6 \times 60 = 360$ sec

$\therefore$ Acceleration, $a = \frac{v-u}{t} = \frac{25-10}{360}$

$= \frac{15}{360}$

$\Rightarrow \boxed{a = \frac{1}{24} m/s^2}$

Distance travelled, $S = ut + \frac{1}{2}at^2$

$= 10 \times 360 + \frac{1}{2} \times \frac{1}{24} \times 360^2$

$= 3600 + \frac{1}{2} \times \frac{1}{24} \times 360 \times 360$

$= 3600 + 2700$

$= 6300$ m

$\boxed{S = 6.3 \text{ Km}}$

3/a) A train, starting from rest, travels with uniform acceleration & acquires a velocity of 60 Km/hr in two minutes. Find the acceleration & the distance travelled in this time.

Soln Initial velocity, $u = 0$

Final velocity, $v = 60$ Km/hr $= 60 \times \frac{1000}{3600} = \frac{50}{3}$ m/s

time $= 2$ minute $= 2 \times 60 = 120$ sec

$\therefore a = \frac{v-u}{t} = \frac{\frac{50}{3} - 0}{120} = \frac{50}{3} \times \frac{1}{120} = \frac{5}{36}$ m/s^2

Distance travelled, $S = ut + \frac{1}{2}at^2$

$= 0 \times 120 + \frac{1}{2} \times \frac{5}{36} \times 120 \times 120$

$= 0 + 1000$

$\Rightarrow S = 1000$ mt

$\Rightarrow \boxed{S = 1 \text{ Km}}$

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3.6 PROJECTILE

A body is projected into the space and is no longer being propelled by fuel is called a projectile.

Some energy is given to the projectile at the initial stage. As it moves through space there is no need of energy to it and it moves freely under the action of gravity.

Example -

a) A bullet fired from a rifle.

b) A cricket ball is thrown into space.

c) A bomb or small bag dropped from an aeroplane.

TERMS RELATED TO PROJECTILE →

1. Maximum Height :→ It is the maximum distance travelled by the projectile in the vertical direction.

2. Time Of Ascent :→ It is the time taken by the projectile to rise the highest point.

3. Time Of Descent :→ It is the time taken by the projectile to come down from the highest point to the point of projection.

4. Horizontal Range :→ It is the distance travelled by the projectile in the horizontal direction.

5. Equation of Trajectory :→ It is an equation connecting the horizontal and vertical distances travelled by the projectile.

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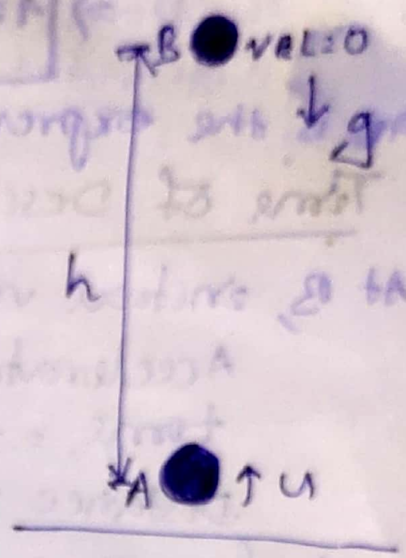
A PROJECTILE FIRED VERTICALLY UPWARDS

Consider a projectile thrown from A with a velocity 'u' in vertically upward direction. Its velocity decreases gradually till it becomes zero at B.



(a) Maximum Height :→

At A, initial velocity = u

At B, Final velocity = 0

Acceleration = −g

Distance travelled = h

Applying the eqn,

$$v^2 - u^2 = 2as$$

$$\Rightarrow 0 - u^2 = 2(-g)h$$

$$\Rightarrow -u^2 = -2gh$$

$$\text{or } u^2 = 2gh$$

$$\Rightarrow \boxed{h = \frac{u^2}{2g}} \quad -(1)$$

This is the expression for Maximum Height.

(b) Time of Ascent :→

At A, initial velocity = u

At B, Final velocity = 0

Acceleration = −g

Time = t.

Applying the eqn,

$$v = u + at$$

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$\Rightarrow u = gt$

$\Rightarrow \boxed{t = \frac{u}{g}}$ — (2)

This is the expression for Time of Ascent.

(c) Time of Descent:→

At B, initial velocity = 0
Acceleration = g
time = t'
Distance travelled = h

Applying the eqn

$$S = ut + \frac{1}{2}gt'^2$$

$$\Rightarrow h = 0 \times t' + \frac{1}{2}gt'^2$$

$$\Rightarrow h = \frac{1}{2}gt'^2$$

$$\Rightarrow t'^2 = \frac{2h}{g}$$

$$\Rightarrow t'^2 = \frac{2}{g} \times \frac{u^2}{2g} \quad (\text{from eqn-1})$$

$$\Rightarrow t'^2 = \frac{u^2}{g^2}$$

$$\Rightarrow \boxed{t' = \frac{u}{g}} \text{ — (3)}$$

This is the expression for time of Descent

From eqn (2) & (3), it is clear that body takes equal time in going up and coming down from a certain height.

(d) Speed On Reaching the ground-

At B, initial velocity = 0
At A, final velocity = v
Acceleration = g
- Distance travelled = h

Applying equation.

$$v^2 - u^2 = 2as$$
$$\Rightarrow v^2 - 0 = 2gh$$
$$\Rightarrow v^2 = 2gh$$
$$\Rightarrow v^2 = 2g \times \frac{u^2}{2g}$$
$$\Rightarrow v^2 = u^2$$
$$\Rightarrow \boxed{v = u}$$

Thus the body fall back with same speed.

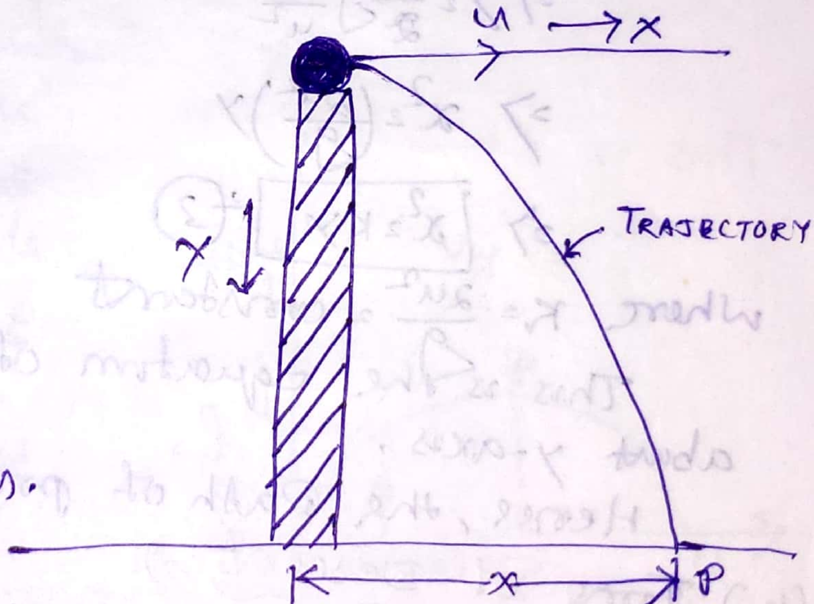
(e) Horizontal Range:→

Since the projection has no component of velocity in the horizontal direction, it travels no distance in the horizontal direction.

$$\therefore \boxed{\text{Range} = \text{zero}}$$

B. PROJECTILE FIRED HORIZONTALLY

Consider a body projected with a velocity in horizontal direction, from a height 'h' above the ground. As it moves through the space it experiences two velocities.



(i) velocity u in horizontal direction →

This is uniform & is imparted to it by agent which projects it.

(ii) velocity in the vertical direction →

This is non-uniform & increases due to 'g'. its initial value is zero.

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Thus the projectile moves under the action of a uniform and non-uniform velocity acting at right angles to each other.

(a) Equation of Trajectory :→

Horizontal equation of motion (uniform)

$$x = ut \quad \text{——} (1)$$

vertical equation of motion (non-uniform)

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow y = 0 \times t + \frac{1}{2}gt^2$$

$$\Rightarrow y = \frac{1}{2}gt^2 \quad \text{——} (2)$$

From eqn-(1) $t = \frac{x}{u}$

Substituting in eqn (2), we get

$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$$

$$\Rightarrow y = \frac{1}{2}g\frac{x^2}{u^2}$$

$$\Rightarrow x^2 = \left(\frac{2u^2}{g}\right)y$$

$$\Rightarrow \boxed{x^2 = ky} \quad \text{——} (3)$$

where $k = \frac{2u^2}{g}$ = constant

This is the equation of parabola, symmetric about y-axes.

Hence, the path of projectile is Parabolic.

(b) Time of Descent :→

Initial velocity = 0

Acceleration = g

time = t

Distance travelled = h

Applying equation

$$S = ut + \frac{1}{2}at^2$$

$\Rightarrow h = 0 \times t + \frac{1}{2} g t^2$

$\Rightarrow h = \frac{1}{2} g t^2$

$\Rightarrow t^2 = \frac{2h}{g}$

$\Rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$ — (4)

(c) Horizontal Range (x)

Since the motion of the projectile in horizontal direction is uniform, horizontal Range is given by

$x = ut$

where $t \rightarrow$ time of descent

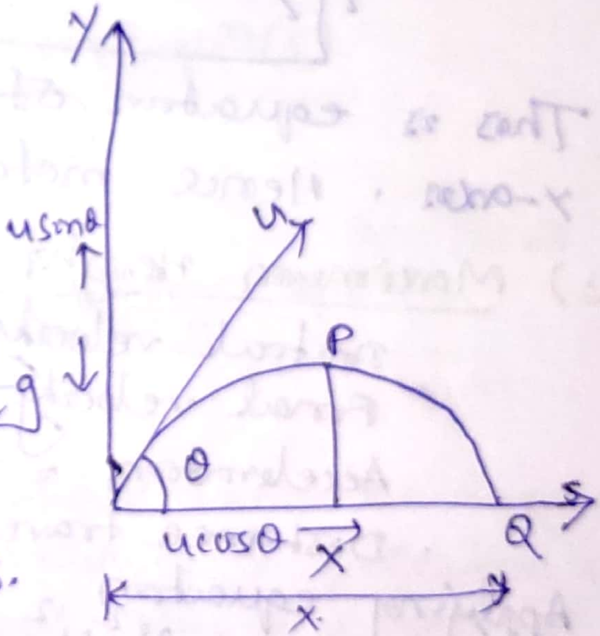
Substituting the value of 't' form eqn (4)

$\boxed{x = u\sqrt{\frac{2h}{g}}}$ — (5)

3.7

C. PROJECTILE FIRED AT AN ANGLE θ WITH HORIZONTAL

Consider a particle fired with a velocity 'u' at an angle 'θ' with the horizontal. The projectile rises to the highest point P and falls back to the point Q lying on the level of projection.



Resolve u in two components.

(i) $u\cos\theta$ along horizontal

This is uniform, since g has no component in this direction.

(ii) $u\sin\theta$ along vertical

This is non-uniform, since g acts opposite to it.

(a) Equation of Trajectory :-

Horizontal equation of motion,

$x =$ velocity $\times$ time

$\Rightarrow x = (u\cos\theta)t$ — (1)

vertical equation of motion

$s = ut + \frac{1}{2}gt^2$

$\Rightarrow y = (u\sin\theta)t + \frac{1}{2}(-g)t^2$

$\Rightarrow y = (u\sin\theta)t - \frac{1}{2}gt^2$ — (2)

From equation - (1)

$$\boxed{t = \frac{x}{u\cos\theta}}$$

Substituting in eqn (2), we get

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2} \text{ — (3)}$$

This is equation of Parabola, symmetric about y-axis. Hence motion of projectile is parabolic.

(b) Maximum Height :-

Initial velocity $= u\sin\theta$

Final velocity $= 0$

Acceleration $= -g$

Distance travelled $= y$

Applying equation

$v^2 - u^2 = 2as$

$\Rightarrow 0 - (u\sin\theta)^2 = 2(-g)y$

$\Rightarrow -u^2\sin^2\theta = -2gy$

$\Rightarrow u^2\sin^2\theta = 2gy$

$$\Rightarrow \boxed{y = \frac{u^2\sin^2\theta}{2g}} \text{ — (4)}$$

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(c) Time of Ascent :→

Initial velocity = $u\sin\theta$

Final velocity = 0

Acceleration = $-g$

Time of Ascent = t

Applying equation of motion,

$$v = u + at$$

$$\Rightarrow 0 = u\sin\theta - gt$$

$$\Rightarrow gt = u\sin\theta$$

$$\Rightarrow \boxed{t = \frac{u\sin\theta}{g}} \quad \text{--- (5)}$$

(d) Total time of flight (T) :→

Since the motion from O to P and P to Q is symmetric w.r.to each other, total time of flight is twice to reach the highest point

$$\therefore \boxed{T = 2t}$$

Substituting the value of 't' from equ^n (5)

$$\therefore \boxed{T = \frac{2u\sin\theta}{g}} \quad \text{--- (6)}$$

(e) Horizontal Range (x)

$$\therefore x = uT$$

$$\Rightarrow x = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2 2\sin\theta \cdot \cos\theta}{g}$$

$$\Rightarrow \boxed{x = \frac{u^2\sin 2\theta}{g}} \quad \text{--- (7)}$$

From equ^n (7), it is clear that Horizontal Range depends upon the value of velocity of projection 'u' and angle of projection θ. Fixed value of u, range can be changed by changing θ.

$\therefore$ Range will be maximum, if $\sin 2\theta$ is maximum

$$\therefore \sin 2\theta = 1 = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ \quad \Rightarrow \boxed{\theta = 45^\circ}$$

This is the condition for Maximum Horizontal Range.

$$\therefore X_{Max} = \frac{u^2}{g} \times 1$$

$$\Rightarrow \boxed{X_{Max} = \frac{u^2}{g}} \quad \text{--- (8)}$$

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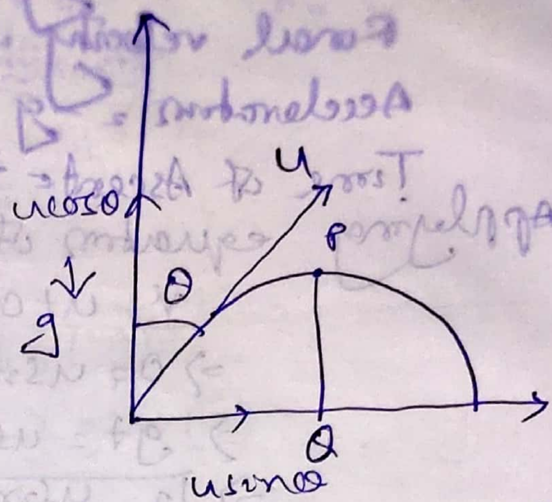
D. PROJECTILE FIRED AT AN ANGLE θ WITH VERTICAL

When a body is projected with a velocity u making an angle θ with the vertical.



(a) $u\sin\theta$ along horizontal

This is uniform, since g has no component along this direction.

(b) $u\cos\theta$ along vertical

This is non-uniform, since g acts opposite to it.

(i) Equation of Trajectory :→

Horizontal equation of motion

$$x = \text{velocity} \times \text{time}$$

$$x = (u\sin\theta)t \quad \text{—(1)}$$

vertical equation of motion

$$S = ut + \frac{1}{2}gt^2$$

$$\Rightarrow y = u\cos\theta \cdot t + \frac{1}{2}(-g)t^2$$

$$= u\cos\theta t - \frac{1}{2}gt^2 \quad \text{—(2)}$$

From eqn (1)

$$t = \frac{x}{u\sin\theta}$$

Substituting in eqn (2), we get

$$y = u\cos\theta \cdot \frac{x}{u\sin\theta} - \frac{1}{2}g\left(\frac{x}{u\sin\theta}\right)^2$$

$$\boxed{y = x\cdot\cot\theta - \frac{1}{2}\frac{g}{u^2\sin^2\theta}x^2} \quad \text{—(3)}$$

This is eqn of parabola, symmetric about y-axis. Hence motion of the projectile is parabola

(2) Time of Ascent —

Initial velocity $= u\cos\theta$

Final velocity $= 0$

Acceleration $= -g$

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Time of Ascent = t

Applying eqn of motion,

$v = u + at$

$\Rightarrow 0 = u\cos\theta + (-g)t$

$\Rightarrow 0 = u\cos\theta - gt$

$\Rightarrow gt = u\cos\theta$

$\Rightarrow \boxed{t = \frac{u\cos\theta}{g}}$ — (4)

ii) Maximum Height :-

Initial velocity = $u\cos\theta$

Final velocity = 0

Acceleration = $-g$

Distance travelled = h

Applying eqn of motion

$v^2 - u^2 = 2as$

$\Rightarrow 0 - (u\cos\theta)^2 = 2(-g)h$

$\Rightarrow -u^2\cos^2\theta = -2gh$

$\Rightarrow u^2\cos^2\theta = 2gh$

$\Rightarrow \boxed{h = \frac{u^2\cos^2\theta}{2g}}$ — (5)

iv) Total time of flight :-

Since the motion from O to P and P to Q is symmetric w.r.to each other, total time of flight is twice taken to reach the highest point

$\therefore T = 2t$

$\Rightarrow \boxed{T = \frac{2u\cos\theta}{g}}$ — (6)

(v) Horizontal Range (x)

$$X = uT$$

$$= u\sin\theta \times \frac{2u\cos\theta}{g}$$

$$\boxed{x = \frac{u^2 \sin 2\theta}{g}} \quad (7)$$

From equn (7), it is clear that horizontal range depends upon the value of velocity of projection 'u' and angle of projection θ. For a fixed value of 'u', range can be changed by changing θ.

∴ Range will be maximum, if sin 2θ is maximum

$$\therefore \sin 2\theta = 1 = \sin 90$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \boxed{\theta = 45^\circ}$$

This is the condition for Maximum Horizontal Range.

$$X_{Max} = \frac{u^2}{g} \times 1$$

$$\Rightarrow \boxed{X_{Max} = \frac{u^2}{g}} \quad (8)$$

Problem-01 A body is thrown vertically upwards with a velocity 19.6 m/s. Calculate

(1) Time of Ascent
(2) Maximum Height
(3) Speed on reaching the ground.

Soln Given u = 19.6 m/s

a) Max. Height $h = \frac{u^2}{2g} = \frac{19.6 \times 19.6}{2 \times 9.8} = 19.6$ m/s

b) Time of Ascent, $t = \frac{u}{g} = \frac{19.6}{9.8} = 2$ sec.

c) Speed on reaching ground $v = u = 19.6$ m/s

Problem-2 A stone is thrown vertically upwards with an initial velocity 14 m/s. Find the maximum height reached and the time of Ascent, $g = 9.8\,m/s^2$.

Soln

Given initial velocity, $u = 14\,m/s$

Final ~~velocity~~ $v = 0$

Acceleration, $g = 9.8\,m/s^2$

We know,

Max. Height $h = \frac{u^2}{2g} = \frac{14^2}{2\times 9.8} = \frac{14\times 14\times 10}{2\times 98} = 10\,m.$

Time of Ascent $t = \frac{u}{g} = \frac{14}{9.8} = \frac{14\times 10}{98} = 1.428$ sec

Problem-3 A body projected vertically upwards reaches a height of 89 m. Calculate its initial velocity.

Soln

We know

Max. Height, $h = \frac{u^2}{2g}$

$\Rightarrow 89 = \frac{u^2}{2\times 9.8}$

$\Rightarrow u^2 = 89\times 19.6$

$\Rightarrow u^2 = 1744.4$

$\Rightarrow \boxed{u = 41.76\,m/s}$

Problem-4 A projectile is fired at an angle 60° with a speed of 30 m/s to the horizontal. Calculate the maximum ~~speed~~ height, Time of flight & horizontal Range.

Soln - Given $u = 30\,m/s$, $\theta = 60°$

Max. Height 'h' $= \frac{u^2 \sin^2\theta}{2g}$

$= \frac{30^2 \times (\sin 60)^2}{2\times 9.8}$

$= \frac{900\times \left(\frac{\sqrt{3}}{2}\right)^2 \times 10}{2\times 98}$

$= \frac{900\times 3\times 10}{2\times 4\times 98} = \frac{3375}{98}$

$\Rightarrow \boxed{h = 34.43\,m}$

Time of flight $= \frac{2u\sin\theta}{g}$

$= \frac{2\times30\times\sin60}{9.8}$

$= \frac{\cancel{2}\times\cancel{30}^{15}\times\sqrt{3}\times10}{2\times\cancel{98}49}$

$= \frac{150\sqrt{3}}{49}$

$\Rightarrow \boxed{T = 5.302\ sec}$

Horizontal Range $= \frac{u^2\sin2\theta}{g}$

$= \frac{30^2\times\sin(2\times60)}{9.8}$

$= \frac{\cancel{900}^{\cancel{450}\,225}\times\sqrt{3}\times10}{2\times\cancel{98}49}$

$= \frac{2250\sqrt{3}}{49}$

$\boxed{x = 79.52m}$

Problem-5 Initial speed of a shell is 392 m/s. At what angle must the gun be fired, if the projectile is to strike the target at same level as the gun and at a distance of 7840 m from it?

Soln Here $u = 392$ m/s

$x = 7840$ m

$\therefore$ we know $x = \frac{u^2\sin2\theta}{g}$

$\Rightarrow 7840 = \frac{392^2\sin2\theta}{9.8}$

$\Rightarrow \sin2\theta = \frac{\cancel{7840}^{2010\,0.7}\times9.8}{\cancel{392}\times\cancel{392}^{\cancel{28}\,14}}$

$\Rightarrow \sin2\theta = \frac{\cancel{10}\times\cancel{7}^{1}}{\cancel{10}\times\cancel{14}^{2}}$

$\Rightarrow \sin2\theta = \frac{1}{2} = \sin30^\circ$

$\Rightarrow 2\theta = 30$

$\Rightarrow \boxed{\theta = 15^\circ}$

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Problem 6 An Athlete throws a Javelin to a maximum distance of 80m. How long is it in air and what height does it rise? Neglect the height of athlete.

Soln Maximum range = 80 m.

Max. Horizontal range, $\theta = 45^\circ$,

$\therefore$ Max. Horizontal range $= \frac{u^2}{g}$

$$\Rightarrow \frac{u^2}{g} = 80$$

$$\Rightarrow u^2 = 80 \times 9.8$$

$$\Rightarrow u^2 = \frac{\cancel{80}^{8} \times 98}{\cancel{10}}$$

$$\Rightarrow u^2 = 784$$

$$\therefore \Rightarrow \boxed{u = 28\ m/s}$$

Total time of flight $T = \frac{2u\sin\theta}{g}$

$$= \frac{2 \times 28 \times \sin 45^\circ}{9.8}$$

$$= \frac{2 \times \cancel{28}^{2} \times 10 \times 1}{\cancel{98}_{7} \times \sqrt{2}}$$

$$= \frac{40}{7\sqrt{2}}$$

$$\Rightarrow \boxed{T = 4.04\ sec}$$

Max. Height $y = \frac{u^2\sin^2\theta}{2g}$

$$= \frac{28^2 (\sin 45)^2}{2 \times 9.8}$$

$$= \frac{\cancel{28}^{2} \times \cancel{28}^{4^2} \times 1 \times 10}{2 \times \cancel{98}_{1} \times \cancel{2}_{1}}$$

$$\Rightarrow \boxed{y = 20\ m}$$

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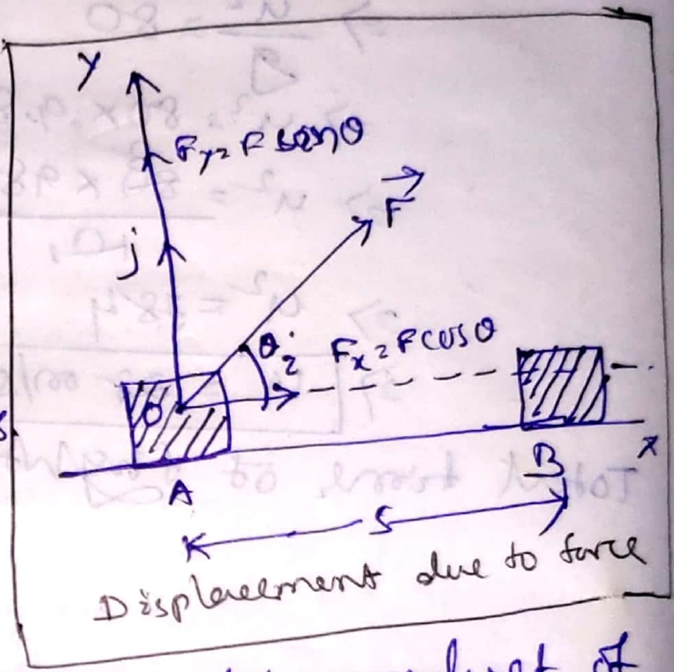
4.1 WORK →

Work is said to be done if a force, acting on a body, displaces the body through a certain distance and the force has some component along the displacement.



Consider a body experiencing a force $\vec{F}$ in a direction inclined at an angle θ with the positive direction of x-axis. Let the body be displaced from A to B through a distance S

$\therefore W = (F\cos\theta)\cdot S$

$= FS\cos\theta$

$\Rightarrow \boxed{W = \vec{F}\cdot\vec{S}}$

So, workdone is defined as the dot product of force and displacement.

Special Case:

(i) Positive work

If $\theta = 0°$, $\cos\theta = 1$

$W = FS\cos\theta$

or $\boxed{W = FS}$

When force and displacement are in same direction, workdone is positive. This work is said to be done upon the body.

Example

(i) A body falling freely under the action of gravity

(ii) A block placed on the table & pulled by a string

(iii) cutting a fruit with knife.

(iv) A car moving forward,

(v) Throwing a stone

(ii) Zero Work

If $\theta = 90^\circ$, $\cos\theta = 0$

$W = FS\cos 90$

$= FS \times 0$

$\Rightarrow \boxed{W = 0}$

When force acts in a direction at right angles to ~~conto~~ the direction of displacement, no work is done.

Example:

(i) A student sitting on a chair and studying a book does no work, since there is no displacement.

(ii) A person carrying a box over his head and walking on a horizontal road does no work since the force and displacement are always $\perp r$ to each other.

(iii) When a body is displaced along a horizontal surface, no work is done by its weight since it is $\perp r$ to the displacement.

(iii) Negative Work:→

If $\theta = 180^\circ$, $\cos\theta = -1$

$\therefore W = FS\cos 180$

$= FS(-1)$

$\Rightarrow \boxed{W = -FS}$

When force and displacements are in opposite directions workdone is negative. This work is done by the body.

Example:

(i) When brakes are applied to a moving vehicle

(ii) When a body is pulled over a horizontal surface

(iii) When two similar charges approach each other they repel each other.

UNIT:

i) SI - Joule

Work is said to be one Joule if a force of 1 newton displaces a body through a distance of 1m along the direction of force.

$1J = 1N \times 1m = 1\,kg \cdot \frac{m}{s^2} \cdot m = 1\,kg\,m^2 s^{-2}$

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(2) CGS - Erg

Work done is said to be one 'erg' if a force of one dyne displaces the body through a distance of 1 cm along the direction of force.

$1\,erg = 1\,dyn \times 1\,cm$

$\Rightarrow 1\,erg = 1\,gm\,cm\,s^{-2}\,cm$

$\Rightarrow 1\,erg = 1\,gm\,cm^2 s^{-2}$

Relation between Joule & Erg

$1J = 1N \times 1m$

$= 10^5\,dyn \times 100\,cm$

$= 10^5 \times 10^2\,dyn\,cm$

$= 10^7\,erg$

4.2 FRICTION

Defn - Whenever an object moves or tends to move, over the surface of another object there is a force acting between the two surfaces in contact. This is known as Force of friction or Friction.

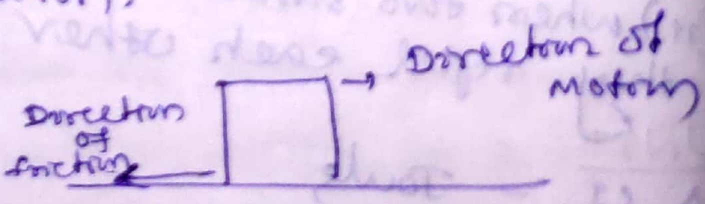
or

Whenever a body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called force of friction.

→ It is always acting in the direction opposite to the direction of motion.

→ It acts along the tangential direction.

→ It is a contact force



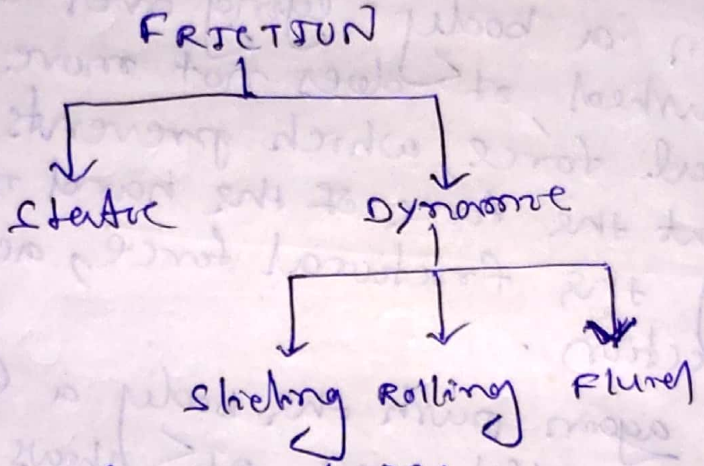
Force of friction depend upon following factors.

(i) Actual Area in contact.

(ii) The nature of the two surfaces in contact with each other

(iii) Normal force with the surfaces are being pressed together.

4.3 TYPES OF FRICTION



Friction is of two types.

(i) Static

(ii) Dynamic

(i) Static Friction -

It is the friction experienced by a body when it is rest or the body tends to move.

→ It is equal & opposite to the applied force.

→ It is a self-adjusting force.

→ It can increase only upto a certain upper limit.

(ii) Dynamic Friction →

It is the friction experienced by a body when it is in motion. It is also called kinetic friction.

It is of three types.

(a) Sliding friction -

It is the friction experienced by a body when it slides over another body.

(b) Rolling friction →
It is the friction experienced by a body when it is rolling over another body.

(c) Fluid Friction →
It is the opposing force which comes into play when a body moves through a fluid.

Limiting friction :→

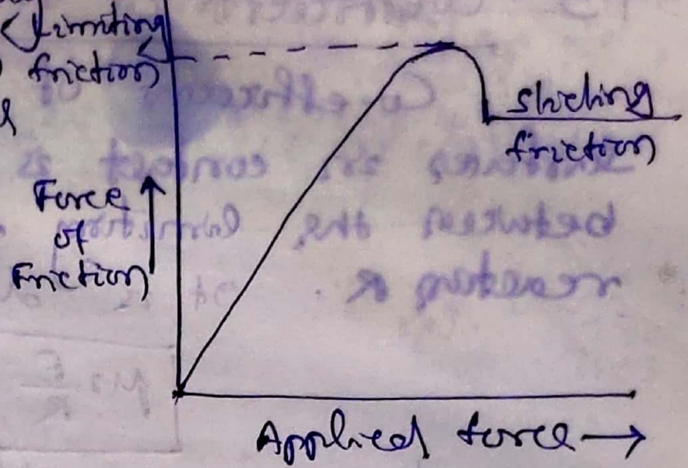
→ When a body lying over another body is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is exactly balanced by the frictional force, acting in the opposite direction.

→ If we again push the body a little harder, it shows that still it is in equilibrium. It shows that frictional force has increased itself so as to become equal and opposite to the applied force.

→ There is a limit beyond which the force of friction can not increase. If the applied force exceeds this limit, the force of friction can not balance it & the body begins to move, in the direction of applied force.

→ Maximum value of force of friction, which comes into play, when a body begins to slide over the surface of other body is known as Limiting friction.

Thus we conclude that when a force is applied on a body at rest, a force of friction called static friction comes into play. This static friction opposes the applied force. On increasing the applied force, static friction



also increases proportionally and reaches a maximum value. This maximum value of static friction is called Limiting Friction. When the applied force is increased beyond the limiting friction, the body begins to slide over the surface on which it was resting. After this, sliding friction acts between the two surfaces.

4.4 LAWS OF LIMITING FRICTION →

Factors upon which the force of limiting friction between two surfaces depends are termed as laws of limiting friction.

(i) The direction of force of friction is always opposite to the direction of motion.

(ii) The force of limiting friction depends upon the nature and state of polish of the surfaces in contact and acts tangentially to the interface between the two surfaces.

(iii) The magnitude of limiting friction 'f' is directly proportional to the magnitude of ~~limiting~~ normal reaction 'R' between the two surfaces in contact i.e

$$F \propto R$$

(iv) The magnitude of the limiting friction between the two surfaces is independent of the area and shape of the surfaces in contact so long as the normal reaction same.

4.5 Coefficient of friction

Co-efficient of friction of a pair of surfaces in contact is defined as the ratio between the limiting friction F to the normal reaction R. It is denoted by μ.

$$\boxed{\mu = \frac{F}{R}}$$

F depends upon the nature and state of polish of the surfaces in contact, μ also depends upon these factors.

4.6 METHODS TO REDUCE FRICTION

There are some ways used to reduce frictions.

1. POLISHING →

When we polish a surface, its roughness decreases. The surface becomes smooth and friction gets reduced.

We sometimes rub the surfaces with a fine sand paper to reduce their roughness.

2. LUBRICATION :→

Oil like substances which when put on a surface, it helps to reduce friction are called Lubricants.

Lubricants can be classified into 3 categories

(i) Liquid like oils (ii) Semi-liquid like grease
(iii) Solid like Talcum Powder.

(i) Liquid like oils →

When few drops of oil are poured in the hinges of a door, the door moves much more smoothly.

when we apply a lubricant between the moving parts of a machine, a thin layer of lubricant is formed between the two surfaces. so that friction is reduced.

(ii) Semi-liquid like Grease :→

Greasing of machines results in less wear & tear. Hence less energy wastage. This helps to increase the efficiency of machines.

In case of Bicycle & motor machine we use grease to increase efficiency.

(iii) Solid like Talcum Powder :→

Sometimes we use solid lubricants in the form of powders. By sprinkling talcum powder on carrom board, the friction between the striker and the board is very much reduced. The striker moves smoothly on the board.

3. BALL - BEARINGS →

We also know that rolling friction is smaller than sliding friction. sliding can be replaced by rolling in most machines, by use of ball bearings.

Ex → we use ball-bearings in shaft of motors, dynamos, axles of vehicles.

4. STREAMLINING :→

As a body is driven through fluid, fluid friction depends upon the shape of the body. of is minimum for a shape known as streamlined shape. This shape is a pin-pointed one. That is why all high speed bodies, aeroplanes, rockets etc have pin-pointed shapes.

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UNIT-5 (GRAVITATION)

5.1 NEWTON'S LAWS OF GRAVITATION

Statement Every Particle of matter in this universe attracts every other particle with a force which varies directly as the product of the masses of two particles & inversely as the square of the distance between them.

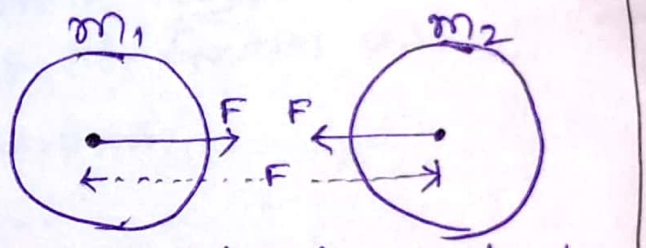
The force of attraction between any two bodies in the universe is known as the Force of gravitation.

EXPLANATION

Consider two bodies A & B of masses m_1 & m_2.

Let 'r' be the distance between their centres.

Let F be the force of attraction between two bodies.



Gravitational attractional between two bodies

According to law of Gravitation

$$F \propto m_1 m_2 \quad \text{—(1)}$$

$$\propto \frac{1}{r^2} \quad \text{—(2)}$$

Combining eqn (1) & (2)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \boxed{F = G \frac{m_1 m_2}{r^2}} \quad \text{—(3)}$$

where $G \rightarrow$ Gravitational Constant

$$\therefore \boxed{G = \frac{F r^2}{m_1 m_2}} \quad \text{—(4)}$$

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Def^n of G

Let $m_1 = m_2 = 1$ unit

$r = 1$ unit

then $G = \frac{F \times 1^2}{1 \times 1}$

$\Rightarrow \boxed{G = F}$

Thus, the gravitational constant may be defined as the magnitude of force of attraction between two bodies each of unit mass & separated by a unit distance from each other.

Unit & value of G

1) In SI

Unit of G is Nm^2kg^{-2}

value of G is $6.67 \times 10^{-11} Nm^2kg^{-2}$

2) In CGS

Unit of G is $dyne\, cm^2 g^{-2}$

value of G is 6.67×10^{-8} dyne $cm^2 gram^{-2}$

Dimension of G

$$G = \frac{Fr^2}{m_1 m_2} = \frac{[M^1L^1T^{-2}][L^2]}{[M^1][M^1]} = [M^{-1}L^3T^{-2}]$$

Dimension of G are -1 in mass, 3 in length & -2 in time respectively.

5.3 GRAVITY - ACCELERATION DUE TO GRAVITY

Gravity means one of the body is earth and 2nd body is either on earth or near it.

Force between earth and a body near it is called gravity.

WEIGHT → The force with which body is attracted towards earth is called weight.

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Thus weight of body & Gravity are similar things.

The acceleration produced by gravity is known as acceleration due to gravity & is denoted by 'g'

5.4 MASS & WEIGHT

MASS	WEIGHT
1. Mass is a measure of amount of matter present in the body	1. Weight is the force acting on an object that pulls it towards the earth
2. Mass of an object is same everywhere.	2. Weight varies from place to place.
3. Mass is a scalar quantity.	3. Weight is a vector quantity.
4. SI Unit of Mass – Kg	4. SI Unit of Weight = Newton
5. Mass of an object can be measured with the help of beam balance.	5. Weight of an object can be measured with the help of spring balance.

5.5. Relation between g and G

According to defination of Gravity

$$F = mg \quad -(1)$$

According to Newton's law of Gravitation

$$F = G\frac{Mm}{R^2} \quad -(2)$$

From eqn -(1) & (2)

$$mg = \frac{GMm}{R^2}$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \quad -(3)$$

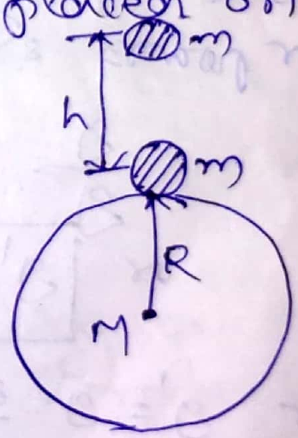
DIFFERENCE BETWEEN g & G

g	G
1. The Acceleration produced on a freely falling body due to gravitational force is known as Acceleration due to gravity.	1. The force of attraction between any two objects of unit masses separated by unit distance in the Universe is known as Universal Gravitational Constant
2. It is denoted by g.	2. It is denoted by G.
3. It changes from place to place.	3. Its value is constant everywhere in the Universe.
4. Its units are m/sec^2	4. Its units are Nm^2/kg^2

5.6 VARIATION OF g

1. ALTITUDE

Consider a body of mass 'm' placed on the surface of earth.



Let M → Mass of earth
R → Radius of earth
g → Acceleration due to gravity on the free surface of earth

Then $g = \frac{GM}{R^2}$ —— (1)

Where G → Gravitational Constant

Suppose the body is taken to a height 'h' above the surface of earth.

Let g' → Acceleration due to gravity at height 'h'

Now eqn (1) is written as

$$g' = \frac{GM}{(R+h)^2} \quad \text{--- (2)}$$

where $(R+h) \rightarrow$ Distance between center of body and center of earth.

Dividing eqn (2) by eqn (1)

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad \text{--- (3)}$$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

Since h is small as compared to R, therefore $\frac{h}{R}$ is very small as compared to 1. So expanding the RHS of above eqn by Binomial theorem and neglecting higher powers & squares of $\frac{h}{R}$ we get

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$\text{or} \quad \boxed{g' = g\left(1 - \frac{2h}{R}\right)} \quad \text{--- (4)}$$

$$\text{or} \quad g' = g - \frac{2gh}{R}$$

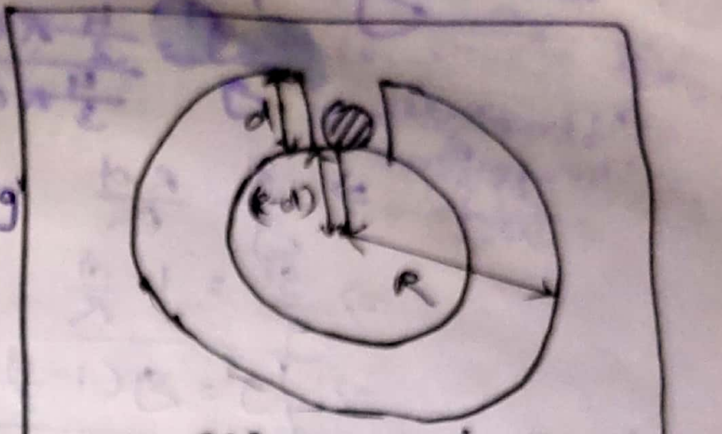
$$\Rightarrow g - g' = \frac{2gh}{R}$$

$$\therefore \boxed{g - g' \propto h}$$

∴ It is clear from above if h increases g' must decrease because g is constant. Thus the value of acceleration due to gravity decreases with increase in height above the surface of earth.

(2) Depth →

Consider a body lying on the surface of earth where the value of acceleration due to gravity 'g'



Effect of depth on 'g'

Let $M \Rightarrow$ Mass of earth
$R \rightarrow$ Radius of earth
$\rho \rightarrow$ Mean density of material of earth.

Assume earth to be a homogeneous sphere.

$$\therefore \quad g = \frac{GM}{R^2}$$

But density $= \frac{\text{Mass}}{\text{Volume}}$ $\Rightarrow$ Mass = volume × density

$$\therefore \quad M = \frac{4}{3}\pi R^3 \times \rho$$

$$\text{so} \quad g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\text{or} \quad \boxed{g = \frac{4}{3}\pi G R \rho} \quad \text{— (1)}$$

Let the body is taken to a depth d below the surface of earth.

Let g^1 be the value of acceleration due to gravity at the depth d.

Now the force of gravity acting on the body is due to inner solid sphere of radius (R-d)

$$\therefore \quad g^1 = \frac{GM^1}{(R-d)^2}$$

where $M^1 \rightarrow$ Mass of inner solid sphere of radius (R-d)

$$\therefore \quad g^1 = \frac{G}{(R-d)^2} \times M^1$$

$$= \frac{G}{(R-d)^2} \times \frac{4}{3}\pi (R-d)^3 \rho$$

$$\therefore \quad \boxed{g^1 = \frac{4}{3}\pi G (R-d) \rho} \quad \text{— (2)}$$

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Dividing equⁿ 2 by equⁿ 1

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R\rho}$$

$$\Rightarrow \frac{g'}{g} = \frac{R-d}{R}$$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{d}{R}$$

$$\Rightarrow \boxed{g' = g\left(1-\frac{d}{R}\right)} \quad (3)$$

This is the equⁿ of variation of g with depth

From equⁿ (3)

$$g' = g - \frac{d}{R}g$$

$$\Rightarrow g - g' = \frac{d}{R}g$$

$$\therefore \boxed{g - g' \propto d} \quad (\because g \,\&\, R \text{ are constant})$$

→ It is clear from above that if d increases, g' must decrease because g is constant. Thus the value of acceleration due to gravity decreases as the depth increase.

Weight of body at the centre of earth-

We know that at a depth d below the free surface of earth

$$g' = g\left(1-\frac{d}{R}\right)$$

At the center of earth d=R.

$$\therefore g' = g\left(1-\frac{R}{R}\right) = 0$$

If m is the mass of body placed at the then weight ⇒ $\boxed{mg' = 0}$

Hence, the weight of the body at the centre of earth is zero.

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5.7 KEEPLER'S LAW OF PLANETORY MOTION →

Copernics first of all introduced the idea that the central body of our planetory system was sun rather than earth. Galileo's construction of a telescope enabled him to discover direct visual evidence supporting "Copernics Theory". Keepler later on conformed. He gave three laws.

(i) Law of Eliptical Orbit →

A planet moves round the sun in an elliptical orbit with sun situated at one of its foci.

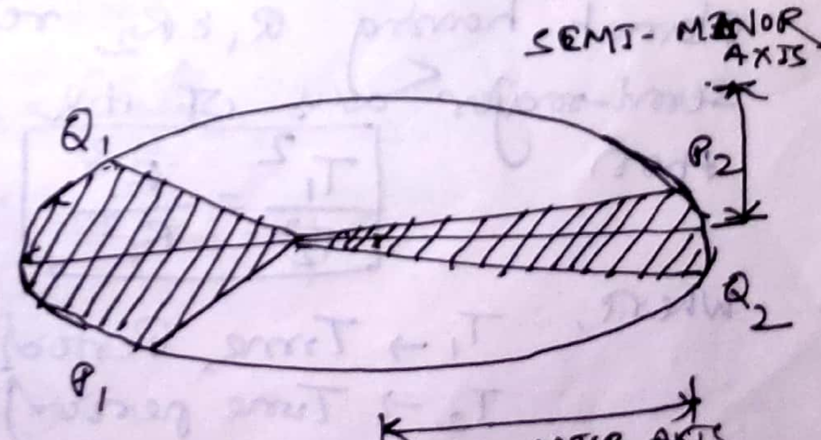
Explanation →

Since the focus of an ellipse is not equidistance from the point on the orbit, the distance of planet from sun varies between a certain minimum and maximum value.

Rotation is the reason for change of season from winter to summer & repeation of the season once in a year.

(ii) Law of Areal Velocities

A planet moves round the sun in such a way that its areal velocity remains constant. (i.e line joining the planet with sun sweeps equal area in equal interval of time)



Rotation of a plant around the sun

Let 't' be time taken by the planet to go from P_1 to Q_1 so that the line SP_1 traverse an area P_1SQ_1 while going from P_2 to Q_2, the planet moves in such a way that

Area P_2SQ_2 = Area P_1SQ_1

Since $SP_2 > SP_1$ $\therefore$ $P_2Q_2 < P_1Q_1$

Since P_1Q_1 & P_2Q_2 are the distances travelled in same time. From this we conclude that the orbital velocity of a planet is not uniform. It is largest when the planet is nearest to the sun & is least when the planet is maximum distance away from sun.

(iii) Law of Time Period (The Harmonic Law)

A planet moves round the sun in such a way that the square of its time period is proportional ~~perpendicular~~ to the cube of semi-major axis of its elliptical orbit.

i.e $\boxed{T^2 \propto R^3}$

- If T_1 & T_2 are the time periods of two planets having R_1 & R_2 respectively as the semi-major axis of the orbits of two planets then

$$\boxed{\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}}$$

Where, $T_1 \rightarrow$ Time Period of Planet-1

$T_2 \rightarrow$ Time period of planet-2

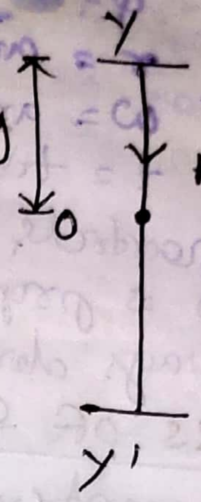
$R_1 \rightarrow$ ~~Radius~~ Semi-major-axis of Planet-1

$R_2 \rightarrow$ Semi-major axis of planet-2

UNIT-6 - OSCILLATIONS AND WAVES

6.1 SIMPLE HARMONIC MOTION (SHM):→

Let a particle be displaced a distance 'y' from the mean position 'O'. A restoring force 'F' tends to bring the particle back to O, due to the property of elasticity.



For smaller Y, the force F is found to be proportional to Y and opposes the increase of y.

$$\therefore \boxed{F = -KY} \quad (1)$$

where K → Force Constant

its value depends on nature of the medium in which the body vibrates.

So, Simple Harmonic motion is the motion in which the restoring force is proportional to displacement from the mean position and opposes the increase.

Acceleration 'a' of the body vibrating in SHM

$$a = \frac{d^2y}{dt^2}$$

According to Newton's 2nd law of motion

F = mass × acceleration

$$\Rightarrow F = m\frac{d^2y}{dt^2} \quad (2)$$

From eqn (1) & (2)

$$\text{or } \frac{d^2y}{dt^2} = -Ky$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} = -\left(\frac{K}{m}\right)y} \quad (3)$$

This eqn is called Differential eqn of SHM

-ve sign is due to acceleration is directed towards mean position.

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Solution of eqn-(3) is

$y = r \sin(\omega t + \phi)$

where ϕ = Phase angle

r = amplitude

ω = angular velocity

t = time period.

Defn → A particle is said to move in SHM if its acceleration is proportional to the displacement and is always directed towards the mean position

PARAMETERS OF SHM →

(i) Displacement(y) :→

Displacement of a particle vibrating in SHM at any instant is defined as its distance from the mean position at that instant.

$$\boxed{y = r \sin \omega t}$$

where r → radius of circle

ω → angular velocity

(ii) Amplitude(r) :→

Amplitude of a particle vibrating in SHM is defined as the maximum displacement on either side of mean position.

From above eqn $\boxed{y = \pm r}$

where r → Amplitude of vibration

(iii) Frequency(n) :→

It is the number of vibrations made by body in one second.

$$\therefore \boxed{n = \frac{1}{T}}$$

(iv) Time Period(T) :→

It is the time taken by particle to complete one vibration.

$$\boxed{T = \frac{2\pi}{\omega}}$$

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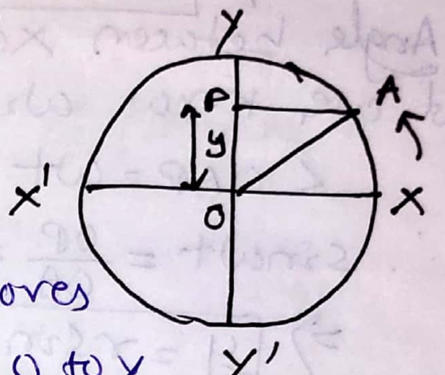
• EXPLANATION OF SHM AS PROJECTION OF A UNIFORM CIRCULAR MOTION ON ANY DIAMETER

Consider a particle 'A' undergoing uniform circular motion in a circle having XOX' & YOY' as the ~~uniform~~ horizontal & vertical ~~diameters~~ diameters respectively.

Let 'P' be the perpendicular drawn from A upon one of the diameters.

P is called 'Projection of A' or 'shadow of A'.



While 'A' is at X, its projection 'P' is at O. As it moves from X to Y, P moves from O to Y along the vertical diameter. As 'A' moves from Y to X', P come back from Y to O. Thus A complete its journey along the circumference of the circle, its projection moves from O to Y, Y to O, O to Y' and Y' to O. If particle A keep on moving projection P keeps on vibrating to and fro about 'O'. Motion of P along YOY' called Simple Harmonic Motion (SHM).

So, Simple Harmonic motion is the motion is defined as the projection of uniform circular motion on the diametre of circle of reference.

6.2. EXPRESSION (FORMULA/EQUATION) FOR DISPLACEMENT, VELOCITY, ACCELERATION OF A BODY/PARTICLE IN SHM

Consider a particle is at A.

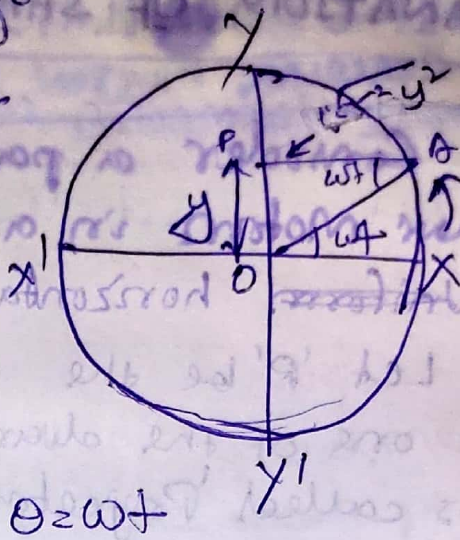
Here P is the position of projection of A, at any instant of time t.

$\angle APO = 90°$

$OP = y$

$OA = r$

In Δ APO, $\angle APO = 90^0$



$\therefore OA^2 = OP^2 + AP^2$

$\Rightarrow r^2 = y^2 + AP^2$

$\Rightarrow AP^2 = r^2 - y^2$

$\Rightarrow \boxed{AP = \sqrt{r^2-y^2}}$

$\theta \rightarrow$ Angle between XO & OA

But we know $\omega = \frac{\theta}{t}$ & $\theta = \omega t$

$\angle OAP = \omega t$

$\therefore \sin\omega t = \frac{OP}{OA} = \frac{y}{r}$

$\Rightarrow \boxed{y = r\sin\omega t}$ — 4

(i) Displacement.

$y = r\sin\omega t$

where $r \rightarrow$ radius of circle

$\omega \rightarrow$ Angular velocity

(ii) Amplitude :→

From eqn it is clear that y is maximum when $\sin\omega t$ is maximum

$\therefore \sin\omega t = \pm 1$

Putting in eqn (4), we get

$y = r(\pm 1)$

$\Rightarrow \boxed{y = \pm r}$

where $r \rightarrow$ Amplitude of Vibration

iii) VELOCITY →

We can get velocity 'v' of the point P by differentiating w.r.t.o time

$$v = \frac{dy}{dt}$$

$$= \frac{d}{dt}(r\sin\omega t)$$

$$= r\cos\omega t \frac{d}{dt}(\omega t)$$

$$= r\omega\cos\omega t$$

$$\Rightarrow \boxed{v = \nu\cos\omega t} \quad (6)$$

where $\nu \to$ Linear Velocity of particle A

In triangle OAP,

$$\cos\omega t = \frac{AP}{OA}$$

$$\Rightarrow \cos\omega t = \frac{\sqrt{r^2-y^2}}{r}$$

$$\therefore v = \nu \cdot \frac{\sqrt{r^2-y^2}}{r}$$

$$= r\omega \frac{\sqrt{r^2-y^2}}{r}$$

$$\Rightarrow \boxed{v = \omega\sqrt{r^2-y^2}} \quad (7)$$

At Mean Position (0)

$y = 0$

$\therefore v = \omega\sqrt{r - 0}$

$\Rightarrow v = r\omega$

$\Rightarrow \boxed{v = \nu}$ (Maximum)

At extreme position ($\pm r$)

$y = \pm r$

$\therefore v = \omega\sqrt{r^2 - r^2}$

$\Rightarrow \boxed{v = 0}$ (minimum)

A particle vibrating in SHM, passes with maximum velocity through mean position and minimum velocity at the extreme position.

iv) **ACCELERATION** –

Acceleration 'a' of the particle is ~~given~~ obtained by differentiating equ^n (6)

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(v\cos\omega t)$$
$$= v\frac{d}{dt}(\cos\omega t)$$
$$= -\omega v \sin\omega t$$
$$= -\omega \cdot r\omega \cdot \frac{y}{r} \quad (\because \sin\omega t = \frac{y}{r})$$
$$\Rightarrow \boxed{a = -\omega^2 y} \text{ — (8)}$$

At Mean

$y = 0$

$\therefore a = -\omega^2 \times 0 = 0$

$\Rightarrow \boxed{a = 0}$ (Min)

At Extreme

$y = \pm r$

$\therefore a = -\omega^2(\pm r)$

$\Rightarrow \boxed{a = \mp\omega^2 r}$ (Max)

A particle vibrating in SHM, has zero acceleration at mean position & Maximum acceleration at Extreme position.

(v) **Time Period** →

It is the time taken by particle to complete vibration,

$$\boxed{T = \frac{2\pi}{\omega}} \text{ — (9)}$$

From eqn (8)

$$\omega^2 = \frac{a}{y}$$
$$\Rightarrow \omega = \sqrt{\frac{a}{y}}$$

Substituting in eqn (9), we get

$$T = \frac{2\pi}{\sqrt{\frac{a}{y}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{y}{a}}} \text{ — (10)}$$

(vi) **Frequency** →

It is the no. of vibrations made by body in 1 s.

$$\therefore \boxed{n = \frac{1}{T}}$$
$$\therefore \boxed{n = \frac{1}{2\pi}\sqrt{\frac{a}{y}}} \text{ — (11)}$$

6.3 Wave Motion

Defn → Wave-motion is the disturbance that travels through the medium & is due to repeated periodic motion of the particles of medium over particle to particle.

TYPES OF WAVE-MOTION

These are two types of wave motion
(i) Transverse Wave-motion
(ii) Longitudinal Wave-motion.

6.4 PROPAGATION OF WAVE MOTION :→

TRANSVERSE WAVE MOTION →

Defn → It is the type of wave motion in which the particles of the medium are vibrating in a direction at right angles to the direction of propagation of wave.

→ Light belongs to this type of wave-motion.
→ A stone thrown in a pond of water information of transverse wave motion. As a transverse wave propagates through the medium, some portion of the medium gets raised above its normal level and is called 'Crest' while some other portion gets depressed below the normal level and is called 'Trough'.

LONGITUDINAL WAVE

Defn → It is the type of wave motion in which the particles of the medium are vibrating in the direction of propagation of wave
→ Sound belongs to this type of wave motion.

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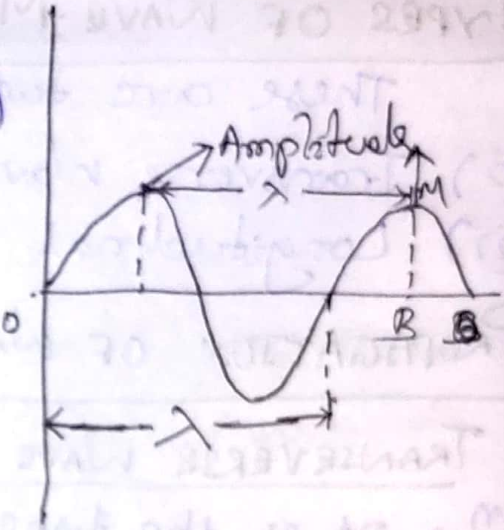
→ A vibrating tuning fork produces longitudinal wave. In such a case, some portion of the medium gets compressed together & is called Compression region while some other portion gets rarefied and is called Rarefraction region.

6.5 WAVE PARAMETERS

(1) TIME PERIOD—(T)

It is the time required to complete one vibration



(2) FREQUENCY — (n)

It is the number of vibrations performed by the particles in one second.

(3) WAVE LENGTH (λ)

The distance travelled by the wave in one time period ie T second is called Wavelength.

(4) AMPLITUDE (r) →

The Maximum displacement of the particle on either side of the mean position is called its amplitude

(5) WAVE NUMBER ($\bar{n}$)

It is defined as the reciprocal of the wavelength of wave.

$$\boxed{\bar{n} = \frac{1}{\lambda}}$$

(6) VELOCITY OF WAVE (v)

It is the distance travelled by the wave during the time (T), a particle completes one vibration.

$$v = \frac{\text{distance}}{\text{time}}$$

or $\boxed{v = \frac{\lambda}{T}}$

6.6 RELATION BETWEEN VELOCITY, FREQUENCY AND WAVE LENGTH OF WAVE

We know that

$$\text{velocity} = \frac{\text{Distance}}{\text{time}}$$

$$\Rightarrow v = \frac{\text{Wave length}}{\text{Time Period}}$$

$$\Rightarrow v = \frac{\lambda}{T}$$

$$\Rightarrow v = \left(\frac{1}{T}\right)\lambda$$

$$\Rightarrow \boxed{v = n\lambda}$$

$\Rightarrow$ velocity of wave = frequency × wavelength

6.7 ULTRASONICS

Defn Sound of frequency greater than the upper limit of audible range (generally 20000 cycles/sec) is termed as ultrasonic.

PROPERTIES →

1. Ultrasonic waves are longitudinal in nature
2. They travel with the speed of sound.
3. These are waves of very high frequency having a range of 2×10^4 to 10^9 Hertz.
4. Propagation of these wave results in the formation of compression and rarefaction.
5. Ultrasonics can constitute narrow beams due to their smaller wavelength.
6. Ultrasonic waves are highly energetic because energy of sound wave is proportional to the square of their frequency.
7. Passage of ultrasonic through a liquid can be used as diffraction grating to produce diffraction of light.

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APPLICATIONS →

1. Thickness Gauging

The process of measuring the thickness of rolled sheets in rolling mills by thickness Gauging

2. Fuel Gauging →

It can be used to make an idea about the fuel present in rockets.

3. Echo Sounding →

It can be used to measure depth of sea

4. Flaw detection →

It can be used to detect internal defects laminations, fissures & other disruptions inside the material without disturbing

5. Drilling holes →

It can be used for drilling holes in hard & brittle materials like aluminium, glass, mica, granite, quartz, ruby, ceramics etc.

6. DIAGNOSTIC USE →

It can be employed in detecting tumors, soft tissue structure, lesions and abnormal growth in the body.

7. BIOLOGICAL EFFECT

These waves are capable of destroying bacteria bacili or small insects. It can be used for sterilisation process.

8. COAGULATION

This method is used for dispersion of fog, removal of smoke from industrial stack, removal of acid fumes etc.

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PROBLEM

1. Find Amount of Heat required to convert 50 gm of ice at $-20^{\circ}C$ to steam at $120^{\circ}C$. Assume specific heat of ice, water, steam as same $= 1\ cal/gm^{\circ}C$

Sol^n The changes goes through five stages, which involves two change of phase

(a) ice at $-20^{\circ}C$ to ice at $0^{\circ}C$
(b) ice at $0^{\circ}C$ to water at $0^{\circ}C$
(c) water at $0^{\circ}C$ to water at $100^{\circ}C$
(d) water at $100^{\circ}C$ to steam at $100^{\circ}C$
(e) steam at $100^{\circ}C$ to steam at $120^{\circ}C$.

(a) Heat required to convert ice at $-20^{\circ}C$ to ice at

$$Q_1 = m S_{ice} (T_2 - T_1)$$
$$= 50 \times 1 (0 - (-20))$$
$$= 50 \times 20$$
$$\Rightarrow \boxed{Q_1 = 1000\ cal}$$

(b) Heat required to convert ice at $0^{\circ}C$ to water at

$$Q_2 = m l_f$$
$$= 50 \times 80$$
$$\Rightarrow \boxed{Q_2 = 4000\ cal}$$

(c) Heat required to convert water at $0^{\circ}C$ to water at $100^{\circ}C$.

$$Q_3 = m S_{water} (T_2 - T_1)$$
$$= 50 \times 1 (100 - 0)$$
$$= 50 \times 100$$
$$\Rightarrow \boxed{Q_3 = 5000\ cal}$$

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(d) Heat required to convert water at 100°C to steam at 100°C.

$Q_4 = m l_v$

$= 50 \times 540$

$\boxed{Q_4 = 27000 \text{ cal}}$

(e) Heat required to convert steam at 100°C to steam at 120°C.

$Q_5 = m S_{steam}(T_2 - T_1)$

$= 50 \times 1 (120 - 100)$

$= 50 \times 20$

$\Rightarrow \boxed{Q_5 = 1000 \text{ cal}}$

Total Amount of Heat Required

$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$= (1000 + 4000 + 5000 + 27000 + 1000) \text{ cal}$

$\boxed{Q = 38000 \text{ cal}}$

Q) Calculate the quantity of heat required to raise the temperature of 10gm of ice at -10°C to water at 60°C. [2018-W(Reg)]

Soln The changes goes through 3 stages, which involves one change of phase

(a) Convert ice at -10°C to ice at 0°C

(b) Convert ice at 0°C to water at 0°C

(c) Convert water at 0°C to water at 60°C

(a) Heat Required to convert ice at -10°C to ice at 0°C

$Q_1 = m S_{ice}(\theta_2 - \theta_1)$

$= 10 \times 1 (0 - (-10))$

$= 10 \times 10$

$\Rightarrow \boxed{Q_1 = 100 \text{ cal}}$

(b) Heat required to convert ice at 0°C to water at 0°C

$Q_2 = m l_f$

$= 10 \times 80$

$\Rightarrow \boxed{Q_2 = 800 \text{ cal}}$

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(c) Heat required to convert water at 0°C to water at 60°C

$Q_3 = m S_{water}(\theta_2 - \theta_1)$

$= 10 \times 1 (60-0)$

$= 10 \times 60$

$\Rightarrow \boxed{Q_3 = 600 \text{ cal}}$

∴ Total Amount of heat required $Q = Q_1 + Q_2 + Q_3$

$\Rightarrow Q = (100 + 800 + 600)$ cal

$\Rightarrow \boxed{Q = 1500 \text{ cal}}$

Q) Find the Amount of heat in Joule required to convert heat 8gm of ice at 0°C to water at 15°C. Given $L_f = 336$ Joules/gm, Specific Heat of water = 4.185 J/gm

Soln The changes goes through 2 stages, which involves one change of phase

(a) ice at 0°C to water at 0°C

(b) water at 0°C to water at 15°C.

(a) Heat required to convert ice at 0°C to water at 0°C

$Q_1 = m L_f$

$= 8 \times 336$

$\boxed{Q_1 = 2688 \text{ Joule}}$

b) Heat required to convert water at 0°C to water at 15°C

$Q_2 = m s \theta$

$= m s (\theta_2 - \theta_1)$

$= 8 \times 4.185 \times (15-0)$

$= 120 \times 4.185$

$\Rightarrow \boxed{Q_2 = 502.2 \text{ Joule}}$

∴ Total Amount of Heat required

$Q = Q_1 + Q_2$

$= 2688 + 502.2$

$\Rightarrow \boxed{Q = 3190.2 \text{ Joule}}$

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Q) A rod measures 1 mt at 10°C. Its length is increased by 4×10^{-4} m at 50°C. Find the volume coefficient of rod.

Soln: $L_{10} = 1\,mt$ $\quad T_1 = 10°C$

$\Delta L = L_{50} - L_{10} = 4\times10^{-4}$ $\quad \boxed{T_2 = 50°C}$

We know $\alpha = \dfrac{\Delta L}{L\,\Delta T}$

$= \dfrac{(L_{50} - L_{10})}{L_{10}(T_2 - T_1)}$

$= \dfrac{4\times10^{-4}}{1(50-10)}$

$= \dfrac{4\times10^{-4}}{4\times10}$

$\boxed{\alpha = 10^{-5}\,{}^{0}C^{-1}}$

$\therefore \gamma = 3\alpha = 3\times10^{-5}\,{}^{0}C^{-1} \Rightarrow \boxed{\gamma = 3\times10^{-5}\,{}^{0}C^{-1}}$

Q) A piece of wire has a length of 2.0 mt at 0°C. Find its length at 100°C. Given $\alpha = 17\times10^{-6}\,{}^{0}C^{-1}$

Soln: $l_0 = 2\,mt$, $t_1 = 0°C$, $t_2 = 100°C$

We know, $\alpha = \dfrac{l_2 - l_1}{l_1(t_2 - t_1)}$

$= \dfrac{l_{100} - l_0}{l_0(t_2 - t_1)}$

$\Rightarrow 17\times10^{-6} = \dfrac{l_{100} - 2}{2\times100}$

$\Rightarrow 34\times10^{-6}\times10^{2} = l_{100} - 2$

$\Rightarrow l_{100} - 2 = 34\times10^{-4}$

$\Rightarrow l_{100} = 2 + 0.0034$

$= \boxed{l_{100} = 2.0034\,mt}$

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UNIT-7- HEAT AND THERMODYNAMICS

7.1 HEAT →

→ It is an agent which produces sensation of warmth.

→ something which flows from body at higher temp. to body at lower temp.

7.2 UNIT OF HEAT

a) Calorie (cal) → It is the amount of heat required to raise the temp. of 1 gram of water through 1°C.

b) Kilocalorie (Kcal) → It is the amount of heat required to raise the temp. of 1 kg of water through 1°C.

c) Joule → It is defined in terms of Mechanical energy & is equal to workdone when a force of 1N acts through 1m. in direction of force.

$$\boxed{1\ \text{Cal} = 4.18\ \text{Joule}}$$

T~~HERMAL~~

7.1 TEMPERATURE →

It is something that measures the intensity of Heat.

7.2 UNIT

Its Unit is kelvin, but it can also be measured celsius and Faherenheit.

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7.1 DIFFERENCE BETWEEN HEAT & TEMPERATURE

HEAT	TEMPERATURE
1. Heat is nothing but the amount of energy in a body.	1. Temperature is something that measures the intensity of heat.
2. Unit of Heat is Joule	2. Unit of Temperature is Kelvin, celsius, Fahrenheit
3. Heat travels from hotter region to cooler region.	3. Temperature rise when heated & falls when cooled.
4. Heat is measured by Calorimeter.	4. Temperature is measured by Thermometer.
5. Heat measures both Kinetic & potential energy contained by molecules in an object	5. Temperature measures average kinetic energy of molecules in substance.
6. Heat is the cause	6. Temperature is the effect
7. Heat is represented by Q.	7. Temp is represented by 'T'

7.3 SPECIFIC HEAT

Different bodies of same mass required different amounts of heat to raise their temperatures to same level.

(i) Greater the mass m of body, greater heat is required to raise its temperature by same amount, so quantity of heat is

$$Q \propto m$$

(ii) Greater heat is required to raise the temp. higher

$$Q \propto \Delta T$$

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Combining eqn we get,

$$Q \propto m \Delta T$$

or $\boxed{Q = C m \Delta T}$

where $C \rightarrow$ Specific heat or specific heat capacity of body

If $m = 1$, $\Delta T = 1^0 C$,

then $\boxed{C = Q}$

So, specific heat is defined as the amount of heat required to raise the temperature of a unit mass of material through $1^0 C$.

UNIT It's unit $\rightarrow$

DIMENSION

$$C = \frac{Q}{M \Delta T}$$

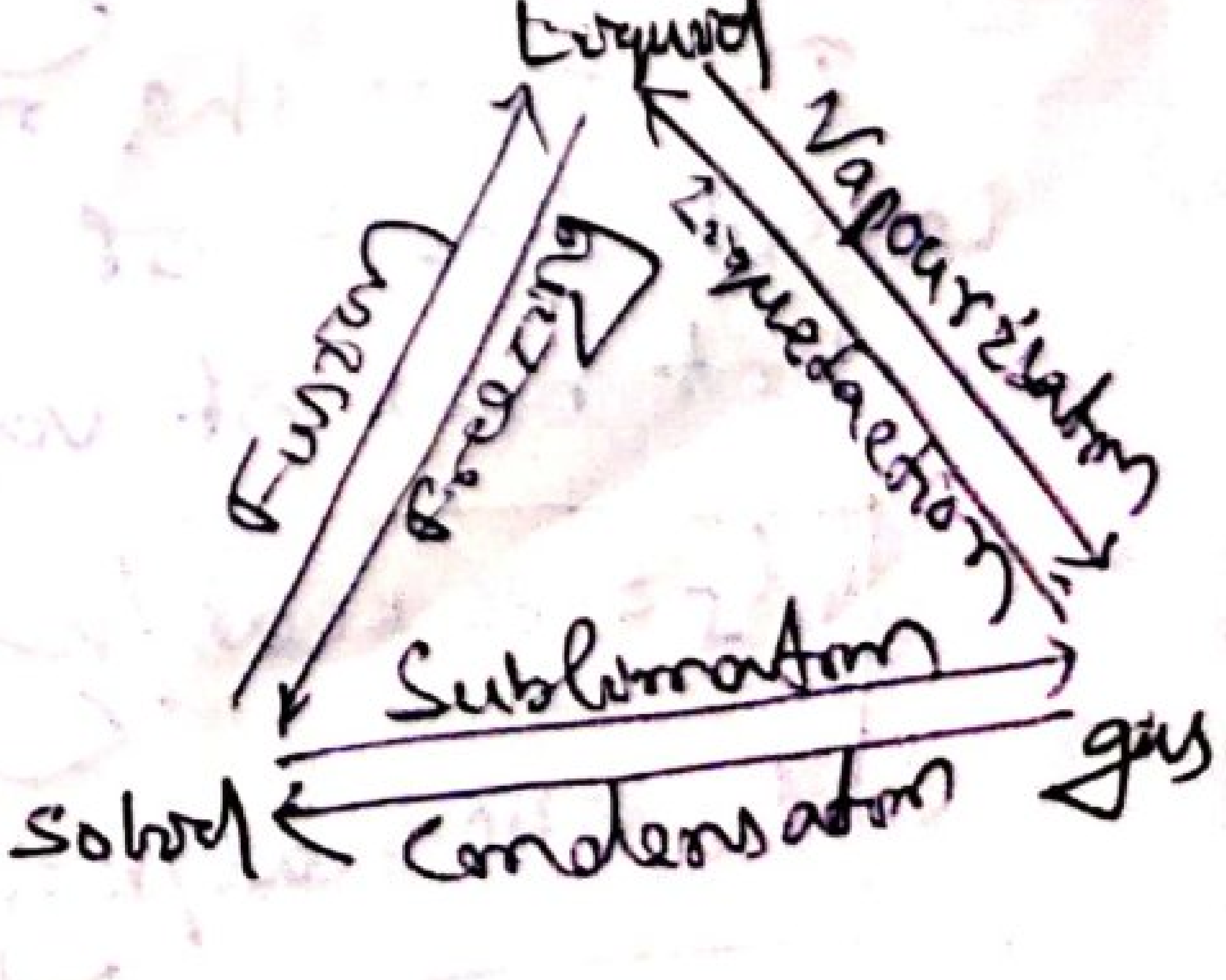
$$= \frac{[M^1 L^2 T^{-2}]}{[M^1][K^1]}$$

$$= [M^0 L^2 T^{-2} K^{-1}]$$

So, the dimensions of specific heat capacity are 0, 2, -2, -1 in mass, length, time and temperature respectively.

7.4 CHANGE OF STATE

We know that ice (solid) gets converted into water (liquid) while water gets converted into steam (gas) on heating. This is true in general for all materials. Different names assigned to various processes of conversion of matter from one state to other



7.9. LATENT HEAT

Whenever a substance undergoes a change in state, whether from solid to liquid or from liquid to gas, it absorbs some quantity of heat without any rise of temp. This heat is called Latent heat.

Specific Latent Heat

If we take a unit mass of substance & change its state, the corresponding latent heat is called "Specific Latent Heat" of substance. Since change of state can take place in two ways.

(i) Specific Latent Heat of fusion (l_f)

It is defined as the amount of ~~substance~~ heat required to convert a unit mass of substance from solid to liquid state at the melting point without any change of temperature.

(ii) Specific Latent Heat of Vapourisation (l_v)

It is defined as the amount of heat required to convert a unit mass of substance from ~~solid~~ liquid to vapour state at the boiling point without any rise of temperature.

If m is the mass of substance

Latent heat of fusion of substance is $L_f = m l_f$

Latent heat of vapourisation of substance is $L_v = m l_v$

$$\boxed{\begin{aligned} l_f &= 80\ kcal/kg = 80\ cal/gm = 335\ J/g = 335000\ J/kg \\ l_v &= 540\ kcal/kg = 540\ cal/gm = 2260\ J/g = 2260000\ J/kg \end{aligned}}$$

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7.5 THERMAL EXPANSION

All solid are found to expand on heating. This is called Thermal Expansion.

It is of three types

a) Linear Expansion
b) Superficial Expansion
c) Cubical Expansion.

a) Linear Expansion →

In this expansion, solid expands in lengthwise.

Let L is the original length of rod.

ΔL is the small increase in length of rod.
ΔT is the raise in temp.

It is found that,

$$\Delta L \propto L$$

$$\propto \Delta T$$

$$\therefore \boxed{\Delta L = \alpha \cdot L \Delta T} \quad -(1)$$

Where $\alpha \rightarrow$ Co-efficient of Linear Expansion of solid.

$$\therefore \boxed{\alpha = \frac{\Delta L}{L \Delta T}} \quad -(2)$$

Unit - $^{\circ}C^{-1}$ or $^{\circ}K^{-1}$

b) Superficial Expansion →

In this expansion, solid expands in Areawise.

Let $S \rightarrow$ Original Surface Area of solid
$\Delta S \rightarrow$ Small increase in Area of solid
$\Delta T \rightarrow$ Raise in temp

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It is found that

$$\Delta S \propto S$$

$$\Delta S \propto \Delta T$$

$$\therefore \boxed{\Delta S = \beta S \Delta T} \quad \text{--- (3)}$$

$$\therefore \boxed{\beta = \frac{\Delta S}{S \Delta T}} \quad \text{--- (4)}$$

Where $\beta \rightarrow$ coefficient of superficial expansion of solid

Unit $^{\circ}C^{-1}$ or $^{\circ}K^{-1}$

c) Cubical expansion of solids →

In this expansion, solid expands in volumewise

Let $V \rightarrow$ Original volume of solid.

$\Delta V \rightarrow$ Small increase in volume of solid

$\Delta T \rightarrow$ Rise in temp.

It is found that,

$$\Delta V \propto V$$

$$\propto \Delta T$$

$$\therefore \Delta V \propto V \Delta T$$

$$\Rightarrow \boxed{\Delta V = \gamma V \Delta T} \quad \text{--- (5)}$$

$$\Rightarrow \boxed{\gamma = \frac{\Delta V}{V \Delta T}} \quad \text{--- (6)}$$

Where $\gamma \rightarrow$ Coefficient of Cubical Expansion of solid

Unit $^{\circ}C^{-1}$ or $^{\circ}K^{-1}$

7.7. Coefficient of Linear, Superficial & Cubical Expansion of solid

a) Coefficient of Linear Expansion (α) →

It is defined as the increase in length per unit length per degree centigrade rise of temperature

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$$\boxed{\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}} \Rightarrow \boxed{\alpha = l_2 - l_1}$$

Unit $^\circ C^{-1}$ or $^\circ K^{-1}$

(b) Coefficient of Superficial Expansion (β) →

It is defined as the change in area of the surface per unit area per $^\circ C$) per degree centigrade rise of temperature.

If $S_0 = 1, t = 1^\circ C$

$$\boxed{\beta = \frac{S_t - S_0}{S_0 t}} \Rightarrow \boxed{\beta = S_t - S_0}$$

Unit $^\circ C^{-1}$ or $^\circ K^{-1}$

(c) Co-efficient of Cubical Expansion (γ) →

It is defined as the change in volume per unit volume at $0^\circ C$, per degree centigrade rise of temperature.

$$\gamma = \frac{V_t - V_0}{V_0 t}$$

If $V_0 = 1, t = 1^\circ C$

then $\boxed{\gamma = V_t - V_0}$

Unit $^\circ C^{-1}$ or $^\circ K^{-1}$

7.8 RELATION BETWEEN α, β & γ

We know

$$\Delta L = \alpha L \Delta T$$

$$\therefore L + \Delta L = L + \alpha L \Delta T \quad \text{— (7)}$$

Similarly, $\Delta V = \gamma V \Delta T$

$V = L^3$ $\quad V + \Delta V = V + \gamma V \Delta T \quad$ — (8)

But

$$V + \Delta V = (L + \Delta L)^3$$

$$= (L + \alpha L \Delta T)^3$$

$$= (L(1 + \alpha \Delta T))^3$$

$$= L^3 (1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$$

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As α is small, α^2 & α^3 term are neglected

$$\therefore V+\Delta V = L^3(1+3\alpha\Delta T)$$

$$\Rightarrow \cancel{V}(1+\gamma\Delta T) = \cancel{V}(1+3\alpha\Delta T)$$

$$\Rightarrow \cancel{1}+\gamma\Delta T = \cancel{1}+3\alpha\Delta T$$

$$\Rightarrow \gamma\cancel{\Delta T} = 3\alpha\cancel{\Delta T}$$

$$\Rightarrow \boxed{\gamma = 3\alpha} \quad \text{— (9)}$$

$$\Rightarrow \boxed{\frac{\alpha}{1} = \frac{\gamma}{3}} \quad \text{— (10)}$$

Again we know,

$$\Delta S = \beta S\Delta T$$

$$\therefore S+\Delta S = S+\beta S\Delta T$$

$$\Delta L = \alpha L\Delta T$$

$$L+\Delta L = L+\alpha L\Delta T$$

we know $S = L^2$

$$\Rightarrow (S+\Delta S) = (L+\Delta L)^2$$

$$= (L(1+\alpha\Delta T))^2$$

$$\Rightarrow (S+\Delta S) = L^2(1+\alpha^2\Delta T^2+2\alpha\Delta T)$$

$$\Rightarrow S+\beta S\Delta T = S(1+\alpha^2\Delta T^2+2\alpha\Delta T)$$

As β is small, β^2 & β^3 terms can be neglected

$$\therefore \cancel{S}(1+\beta\Delta T) = \cancel{S}(1+2\alpha\Delta T)$$

$$\Rightarrow \cancel{1}+\beta\Delta T = \cancel{1}+2\alpha\Delta T$$

$$\Rightarrow \beta\cancel{\Delta T} = 2\alpha\cancel{\Delta T}$$

$$\Rightarrow \boxed{\beta = 2\alpha} \quad \text{— (11)}$$

$$\Rightarrow \boxed{\frac{\alpha}{1} = \frac{\beta}{2}} \quad \text{— (12)}$$

From eqn (10) & (12), it is clear that

$$\boxed{\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}} \quad \text{— (13)}$$

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7.9 HEAT →

It is the energy transferred between substances (systems) due to temperature difference between them.

WORK → The workdone by the system or on the system depends not only the initial & final states of the system but also depends upon the path followed/adopted for the process.

7.10 JOULE'S MECHANICAL EQUIVALENT OF HEAT

It is the relationship between mechanical equivalent of workdone and amount of heat produced. After performing experiment, scientist J.P. Joule found that if a certain amount of work is wholly converted into work, then heat produced.

so $W \propto H$

or $\boxed{W = JH}$

Where $J \rightarrow$ Joules Mechanical Equivalent of Heat

So Mechanical Equivalent of Heat is defined as amount of workdone to produce unit quantity of heat.

$J = 4.18$ Joules/calories

$= 4180$ Joules/kcalories

$= 4.18 \times 10^7$ erg/calories

$= 4.2 \times 10^7$ erg/calories

Ex – Heat Engine, Refrigerator

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According to J. P. Joule, the rule is,

"Whenever heat is converted into work or work into heat, the quantity of energy disappearing in one form is equivalent to the quantity of energy appearing in the other."

7.1) FIRST LAW OF THERMODYNAMICS

Consider some gas enclosed in a barrel having insulating walls and conducting bottom. Let an amount of heat 'Q' be added to the system through the bottom.

If 'U_1' is the internal energy of the system

So Total energy of the system in beginning $= U_1 + Q$

After gaining heat the gas tends to expand, pushing the position from A to B. As a result of this some work 'W' is done by the gas. The work is external work, since the system undergoes a displacement.

If 'U_2' is the final Internal energy of the system

Total Energy of the system at the end $= U_2 + W$

According to law of conservation of energy

$$U_1 + Q = U_2 + W$$

$$\Rightarrow Q = (U_2 - U_1) + W$$

When infinitesimal amount of heat 'dQ' is added to the system, corresponding change in internal energy ~~of the~~ 'dU' & external workdone 'dW' are also small.

then $\boxed{dQ = dU + dW}$ —(1)

where Q → Quantity of heat
U → Internal energy of the system.
W → External workdone.

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Statement:-
If the quantity of heat supplied to a system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of increase in internal energy of the system and external workdone by it.

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PROBLEM

1. Find Amount of Heat required to convert 50 gm of ice at $-20^\circ C$ to steam at $120^\circ C$. Assume specific heat of ice, water, steam as same $= 1\ cal/gm^\circ C$

Sol^n The changes goes through five stages, which involves two change of phase

(a) ice at $-20^\circ C$ to ice at $0^\circ C$

(b) ice at $0^\circ C$ to water at $0^\circ C$

(c) water at $0^\circ C$ to water at $100^\circ C$

(d) water at $100^\circ C$ to steam at $100^\circ C$

(e) steam at $100^\circ C$ to steam at $120^\circ C$.

(a) Heat required to convert ice at $-20^\circ C$ to ice at

$$Q_1 = m S_{ice} (T_2 - T_1)$$
$$= 50 \times 1 (0-(-20))$$
$$= 50 \times 20$$
$$\Rightarrow \boxed{Q_1 = 1000\ cal}$$

(b) Heat required to convert ice at $0^\circ C$ to water at

$$Q_2 = m l_f$$
$$= 50 \times 80$$
$$\Rightarrow \boxed{Q_2 = 4000\ cal}$$

(c) Heat required to convert water at $0^\circ C$ to water at $100^\circ C$.

$$Q_3 = m S_{water} (T_2 - T_1)$$
$$= 50 \times 1 (100-0)$$
$$= 50 \times 100$$
$$\Rightarrow \boxed{Q_3 = 5000\ cal}$$

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(d) Heat required to convert water at 100°C to steam at 100°C.

$Q_4 = m l_v$

$= 50 \times 540$

$\Rightarrow \boxed{Q_4 = 27000 \text{ cal}}$

(e) Heat required to convert steam at 100°C to steam at 120°C.

$Q_5 = m S_{steam} (T_2 - T_1)$

$= 50 \times 1 \, (120 - 100)$

$= 50 \times 20$

$\Rightarrow \boxed{Q_5 = 1000 \text{ cal}}$

Total Amount of Heat Required

$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$= (1000 + 4000 + 5000 + 27000 + 1000) \text{ cal}$

$\boxed{Q = 38000 \text{ cal}}$

Q) Calculate the quantity of heat required to raise the temperature of 10gm of ice at -10°C to water at 60°C. [2018-W(Reg)]

Soln The changes goes through 3 stages, which involves one change of phase

(a) Convert ice at -10°C to ice at 0°C
(b) Convert ice at 0°C to water at 0°C
(c) Convert water at 0°C to water at 60°C

(a) Heat Required to convert ice at -10°C to ice at 0°C

$Q_1 = m S_{ice} (\theta_2 - \theta_1)$

$= 10 \times 1 \, (0 - (-10))$

$= 10 \times 10$

$\Rightarrow \boxed{Q_1 = 100 \text{ cal}}$

(b) Heat required to convert ice at 0°C to water at 0°C

$Q_2 = m l_f$

$= 10 \times 80$

$\Rightarrow \boxed{Q_2 = 800 \text{ cal}}$

(c) Heat required to convert water at 0°c to water at 60°c

$Q_3 = m S_{water} (\theta_2 - \theta_1)$

$= 10 \times 1 (60 - 0)$

$= 10 \times 60$

$\Rightarrow \boxed{Q_3 = 600 \text{ cal}}$

∴ Total Amount of heat required $Q = Q_1 + Q_2 + Q_3$

$\Rightarrow Q = (100 + 800 + 600) \text{ cal}$

$\Rightarrow \boxed{Q = 1500 \text{ cal}}$

Q) Find the Amount of heat in Joule required to ~~convert~~ heat 8gm of ice at 0°c to water at 15°C.
Given L_f = 336 Joules/gm, Specific Heat of water = 4.185 J/gm

Soln The changes goes through 2 stages, which involves one change of phase

(a) ice at 0°c to water at 0°c

(b) water at 0°c to water at 15°c.

(a) Heat required to convert ice at 0°c to water at 0°c

$Q_1 = m l_f$

$= 8 \times 336$

$\boxed{Q_1 = 2688 \text{ Joule}}$

b) Heat required to convert water at 0°c to water at 15°c

$Q_2 = m s \theta$

$= m s (\theta_2 - \theta_1)$

$= 8 \times 4.185 \times (15 - 0)$

$= 120 \times 4.185$

$\Rightarrow \boxed{Q_2 = 502.2 \text{ Joule}}$

∴ Total Amount of Heat required

$Q = Q_1 + Q_2$

$= 2688 + 502.2$

$\Rightarrow \boxed{Q = 3190.2 \text{ Joule}}$

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A rod measures 1 mt at 10°C. Its length is increased by 4×10^{-4} m at 50°C. Find the volume coefficient of rod.

Soln: $L_{10} = 1$ mt $T_1 = 10°C$

$\Delta L = L_{50} - L_{10} = 4\times10^{-4}$ $T_2 = 50°C$

We know $\alpha = \frac{\Delta L}{L \Delta T}$

$= \frac{(L_{50} - L_{10})}{L_{10}(T_2 - T_1)}$

$= \frac{4\times10^{-4}}{1(50-10)}$

$= \frac{4\times10^{-4}}{4\times10}$

$\boxed{\alpha = 10^{-5}\ °C^{-1}}$

$\therefore \gamma = 3\alpha = 3\times10^{-5}\ °C^{-1} \Rightarrow \boxed{\gamma = 3\times10^{-5}\ °C^{-1}}$

A piece of wire has a length of 2.0 mt at 0°C. Find its length at 100°C. Given $\alpha = 17\times10^{-6}\ °C^{-1}$

Soln: $l_0 = 2$ mt, $t_1 = 0°C$, $t_2 = 100°C$

We know, $\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}$

$= \frac{l_{100} - l_0}{l_0(t_2 - t_1)}$

$\Rightarrow 17\times10^{-6} = \frac{l_{100} - 2}{2\times100}$

$\Rightarrow 34\times10^{-6}\times10^{2} = l_{100} - 2$

$\Rightarrow l_{100} - 2 = 34\times10^{-4}$

$\Rightarrow l_{100} = 2 + 0.0034$

$\boxed{l_{100} = 2.0034 \text{ mt}}$

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UNIT-7- HEAT AND THERMODYNAMICS

7.1 HEAT →

→ It is an agent which produces sensation of warmth.

→ something which flows from body at higher temp. to body at lower temp.

7.2 UNIT OF HEAT

a) Calorie (cal) → It is the amount of heat required to raise the temp. of 1 gram of water through 1°C.

b) Kilocalorie (Kcal) → It is the amount of heat required to raise the temp. of 1 kg of water through 1°C.

c) Joule → It is defined in terms of Mechanical energy & is equal to workdone when a force of 1N acts through 1m. in direction of force.

$$\boxed{1 \text{ Cal} = 4.18 \text{ Joule}}$$

~~THERMAL~~

7.1 TEMPERATURE →

It is something that measures the intensity of Heat.

7.2 UNIT

Its Unit is kelvin, but it can also be measured celsius and Faherenheit.

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7.1 DIFFERENCE BETWEEN HEAT & TEMPERATURE

HEAT	TEMPERATURE
1. Heat is nothing but the amount of energy in a body.	1. Temperature is something that measures the intensity of heat.
2. Unit of Heat is Joule	2. Unit of Temperature is kelvin, celsius, Fahrenheit
3. Heat travels from hotter region to cooler region.	3. Temperature rise when heated & falls when cooled.
4. Heat is measured by Calorimeter.	4. Temperature is measured by Thermometer.
5. Heat measures both kinetic & potential energy contained by molecules in an object	5. Temperature measures average kinetic energy of molecules in substance.
6. Heat is the cause	6. Temperature is the effect
7. Heat is represented by Q.	7. Temp is represented by 'T'

7.3 SPECIFIC HEAT

Different bodies of same mass required different amounts of heat to raise their temperatures to same level.

(i) Greater the mass 'm' of body, greater heat is required to raise its temperature by same amount, so quantity of heat is

$$Q \propto m$$

(ii) Greater heat is required to raise the temp. higher

$$Q \propto \Delta T$$

Combining eqn we get,

$$Q \propto m \Delta T$$

or $\boxed{Q = c m \Delta T}$

where $c \rightarrow$ Specific heat or specific heat capacity of body

If $m = 1$, $\Delta T = 1^\circ C$,

then $\boxed{c = Q}$

So, specific heat is defined as the amount of heat required to raise the temperature of a unit mass of material through $1^\circ C$.

UNIT It's unit $\rightarrow$

DIMENSION

$$c = \frac{Q}{m \Delta T}$$

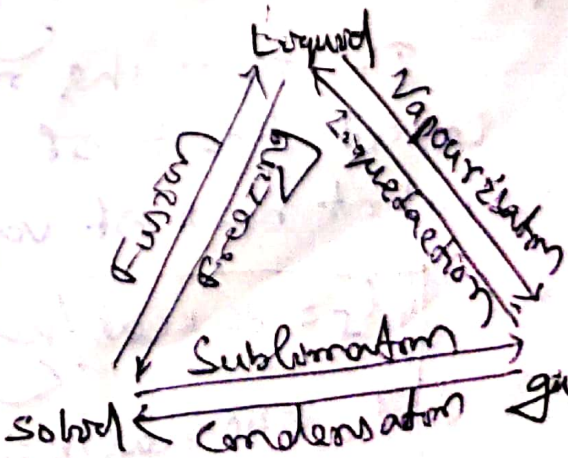
$$= \frac{[M^1 L^2 T^{-2}]}{[M^1][K^1]}$$

$$= [M^0 L^2 T^{-2} K^{-1}]$$

So, the dimensions of specific heat capacity are 0, 2, −2, −1 in mass, length, time and temperature respectively.

7.4 CHANGE OF STATE

We know that ice (solid) gets converted into water (liquid) while water gets converted into steam (gas) on heating. This is true in general for all materials. Different names assigned to various processes of conversion of matter from one state to other



7.9. LATENT HEAT

Whenever a substance undergoes a change in state, whether from solid to liquid or from liquid to gas, it absorbs some quantity of heat without any rise of temp. This heat is called Latent heat.

Specific Latent Heat

If we take a unit mass of substance & change its state, the corresponding latent heat is called "Specific Latent Heat" of substance. Since change of state can take place in two ways:

(i) Specific Latent Heat of fusion (l_f)

It is defined as the amount of ~~substance~~ heat required to convert a unit mass of substance from solid to liquid state at the melting point without any change of temperature.

(ii) Specific Latent Heat of Vapourisation (l_v)

It is defined as the amount of heat required to convert a unit mass of substance from ~~[illegible]~~ liquid to vapour state at the boiling point without any rise of temperature.

If m is the mass of substance.
Latent heat of fusion of substance is $L_f = m l_f$
Latent heat of vapourisation of substance is $L_v = m l_v$

$$\boxed{\begin{aligned} l_f &= 80\ kcal/kg = 80\ cal/gm = 335\ J/g = 335000\ J/kg \\ l_v &= 540\ kcal/kg = 540\ cal/gm = 2260\ J/g = 2260000\ J/kg \end{aligned}}$$

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7.5 THERMAL EXPANSION

All solid are found to expand on heating. This is called Thermal Expansion.

It is of three types.

a) Linear Expansion

b) Superficial Expansion

c) Cubical Expansion.

a) Linear Expansion →

In this expansion, solid expands in lengthwise.

Let L is the Original length of rod.

ΔL is the small increase in length of rod.

ΔT is the raise in temp.

It is found that,

$\Delta L \propto L$

$\propto \Delta T$

$$\therefore \boxed{\Delta L = \alpha \cdot L \Delta T} \quad -(1)$$

Where $\alpha \rightarrow$ Co-efficient of Linear Expansion of solid.

$$\therefore \boxed{\alpha = \frac{\Delta L}{L \Delta T}} \quad -(2)$$

Unit - $^\circ C^{-1}$ or $^\circ K^{-1}$

b) Superficial expansion →

In this Expansion, solid expands in Areawise.

Let $S \rightarrow$ Original Surface Area of solid

$\Delta S \rightarrow$ Small increase in Area of solid

$\Delta T \rightarrow$ Raise in temp.

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It is found that

$$\Delta S \propto S$$

$$\Delta S \propto \Delta T$$

$$\therefore \boxed{\Delta S = \beta S \Delta T} \quad -(3)$$

$$\therefore \boxed{\beta = \frac{\Delta S}{S \Delta T}} \quad -(4)$$

Where $\beta \rightarrow$ coefficient of superficial expansion of solid

Unit $^{o}C^{-1}$ or $^{o}K^{-1}$

c) Cubical expansion of solids →

In this expansion, solid expands in volume wise

Let $V \rightarrow$ Original volume of solid.

$\Delta V \rightarrow$ Small increase in volume of solid

$\Delta T \rightarrow$ Raise in temp.

It is found that,

$$\Delta V \propto V$$

$$\propto \Delta T$$

$$\therefore \Delta V \propto V \Delta T$$

$$\Rightarrow \boxed{\Delta V = \gamma V \Delta T} \quad -(5)$$

$$\Rightarrow \boxed{\gamma = \frac{\Delta V}{V \Delta T}} \quad -(6)$$

Where $\gamma \rightarrow$ Coefficient of cubical expansion of solid

Unit $^{o}C^{-1}$ or $^{o}K^{-1}$

7.7. Coefficient of Linear, Superficial & Cubical Expansion of solid

a) Coefficient of Linear Expansion (α) →

It is defined as the increase in length per unit length per degree centigrade rise of temperature

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$$\boxed{\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}} \Rightarrow \boxed{\alpha = l_2 - l_1}$$

Unit $^\circ C^{-1}$ or $^\circ K^{-1}$

(b) Coefficient of Superficial Expansion (β) →

It is defined as the change in area of the surface per unit area per $^\circ C$ per degree centigrade rise of temperature.

If $S_0 = 1$, $t = 1^\circ C$

$$\boxed{\beta = \frac{S_t - S_0}{S_0 t}} \Rightarrow \boxed{\beta = S_t - S_0}$$

Unit $^\circ C^{-1}$ or $^\circ K^{-1}$

(c) Co-efficient of Cubical Expansion (γ) →

It is defined as the change in volume per unit volume at $0^\circ C$, per degree centigrade rise of temperature.

$$\gamma = \frac{V_t - V_0}{V_0 t}$$

If $V_0 = 1$, $t = 1^\circ C$

then $\boxed{\gamma = V_t - V_0}$

Unit

$^\circ C^{-1}$ or $^\circ K^{-1}$

7.8 RELATION BETWEEN α, β & γ

We know

$$\Delta L = \alpha L \Delta T$$

$$\therefore L + \Delta L = L + \alpha L \Delta T \quad \text{— (7)}$$

Similarly,

$$\Delta V = \gamma V \Delta T$$

$$V = L^3 \quad V + \Delta V = V + \gamma V \Delta T \quad \text{— (8)}$$

But

$$V + \Delta V = (L + \Delta L)^3$$

$$= (L + \alpha L \Delta T)^3$$

$$= (L(1 + \alpha \Delta T))^3$$

$$= L^3 (1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$$

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As α is small, α^2 & α^3 term are neglected

$$\therefore V+\Delta V = L^3(1+3\alpha\Delta T)$$

$$\Rightarrow \cancel{V}(1+\gamma\Delta T) = \cancel{V}(1+3\alpha\Delta T)$$

$$\Rightarrow \cancel{1}+\gamma\Delta T = \cancel{1}+3\alpha\Delta T$$

$$\Rightarrow \gamma\cancel{\Delta T} = 3\alpha\cancel{\Delta T}$$

$$\Rightarrow \boxed{\gamma = 3\alpha} \quad -(9)$$

$$\Rightarrow \boxed{\frac{\alpha}{1} = \frac{\gamma}{3}} \quad -(10)$$

Again we know,

$$\Delta S = \beta S\Delta T$$

$$\therefore S+\Delta S = S+\beta S\Delta T$$

$$\Delta L = \alpha L\Delta T$$

$$L+\Delta L = L+\alpha L\Delta T$$

we know $S = L^2$

$$\Rightarrow (S+\Delta S) = (L+\Delta L)^2$$

$$= (L(1+\alpha\Delta T))^2$$

$$\Rightarrow (S+\Delta S) = L^2(1+\alpha^2\Delta T^2+2\alpha\Delta T)$$

$$\Rightarrow S+\beta S\Delta T = S(1+\alpha^2\Delta T^2+2\alpha\Delta T)$$

As β is small, β^2 & β^3 term can be neglected

$$\therefore \cancel{S}(1+\beta\Delta T) = \cancel{S}(1+2\alpha\Delta T)$$

$$\Rightarrow \cancel{1}+\beta\Delta T = \cancel{1}+2\alpha\Delta T$$

$$\Rightarrow \beta\cancel{\Delta T} = 2\alpha\cancel{\Delta T}$$

$$\Rightarrow \boxed{\beta = 2\alpha} \quad -(11)$$

$$\Rightarrow \boxed{\frac{\alpha}{1} = \frac{\beta}{2}} \quad -(12)$$

From eqn (10) & (12), it is clear that

$$\boxed{\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}} \quad -(13)$$

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It is the relationship between mechanical equivalent of workdone and amount of heat produced. After performing experiment, scientist J.P. Joule found that if a certain amount of work is wholly converted into work, then heat produced.

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or $\boxed{W = JH}$

where $J \rightarrow$ Joules Mechanical Equivalent of Heat

so Mechanical Equivalent of Heat is defined as - amount of workdone to produce unit quantity of heat.

$J = 4.18$ Joules/calories

$= 4180$ Joules/kcalories

$= 4.18 \times 10^7$ erg/calories

$= 4.2 \times 10^7$ erg/calories

Ex - Heat Engine, Refrigerator

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According to J.P. Joule, the rule is,

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7.1) FIRST LAW OF THERMODYNAMICS

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If U_1 is the internal energy of the system

So Total energy of the system in beginning $= U_1 + Q$

After gaining heat the gas tends to expand, pushing the piston from A to B. As a result of this some work 'W' is done by the gas. The work is external work, since the system undergoes a displacement.

If U_2 is the final Internal energy of the system

Total Energy of the system at the end $= U_2 + W$

According to law of conservation of energy

$$U_1 + Q = U_2 + W$$

$$\Rightarrow Q = (U_2 - U_1) + W$$

When infinitesimal amount of heat 'dQ' is added to the system, corresponding change in internal energy 'dU' & external workdone 'dW' are also small.

then $\boxed{dQ = dU + dW}$ —(1)

where Q → Quantity of heat

U → Internal energy of the system.

W → External workdone.

statement :-

If the Quantity of heat supplied to a system is capable of doing work, then the quantity of heat absorbed by the system is equal to the sum of increase in internal energy of the system and external workdone by it.

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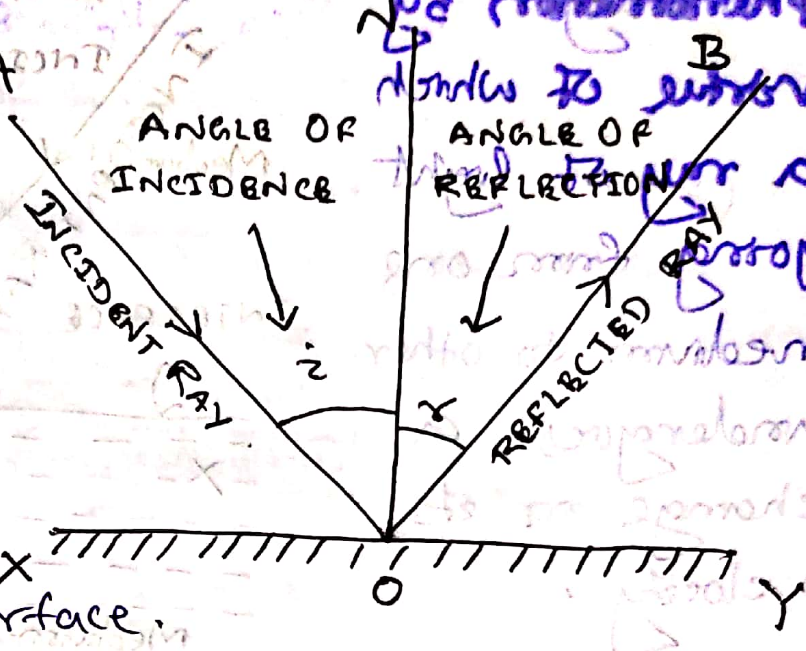
UNIT-8 – OPTICS

8.1 REFLECTION & REFRACTION →

REFLECTION →



It is the property of light by virtue of which, light is sent back to the same medium from which it is comming after being obstructed by a surface.

8.2 Laws Of Reflection →

Phenomenon of reflection is governed by two laws called Laws of reflection.

(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in one plane and that plane is ⊥r to the reflecting surface.

(ii) The angle of incidence is equal to the angle of reflection i.e $\angle i = \angle r$

8.1 REFRACTION

As a ray of light goes from medium 1 (velocity of light v_1) to medium 2 (velocity of light v_2), its velocity changes on crossing the interface XY. The phenomenon is called Refraction.

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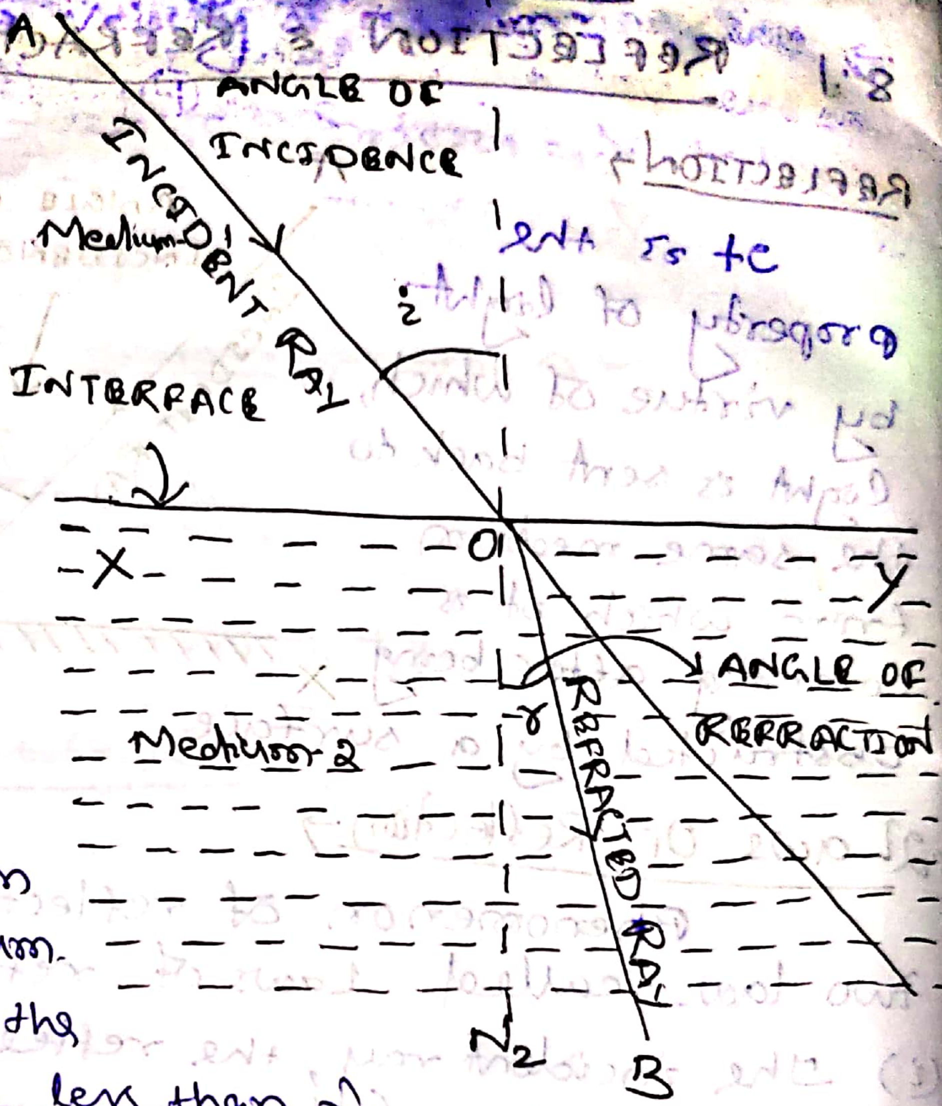
→ It is the phenomenon by virtue of which a ray of light going from one medium to other undergoes a change in its velocity.



→ If light travels from Rarer medium to a denser medium it bends towards the normal i.e r is less than i.

→ If light travels from Denser medium to rarer medium it bends away from the normal i.e r is greater than i.

2 Laws Of Refraction →

Phenomenon of refraction is governed by two laws called laws of refraction.

(i) The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in one plane & that plane is ⊥r to the interface which separates two medium.

(ii) The sine of angle of incidence bears a constant ratio with the sine of angle of refraction.

i.e $\boxed{\frac{\sin i}{\sin r} = \mu \text{(constant)}}$ —(1)

The law is called Snell's law.

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8.3 REFRACTIVE INDEX →

→ The Refractive index of the medium is defined as the velocity of light in vaccum to the velocity of light in the medium.

→ It is represented by $\mu = \frac{{}^{1}\mu}{1} \times \frac{1}{{}^{2}\mu}$

$$\therefore \boxed{\mu = \frac{c}{v}} \quad -② $$

where $\mu \rightarrow$ Refractive index of the medium
$c \rightarrow$ Velocity of light in that medium
$v \rightarrow$ Velocity of light in vaccum.

From eqn-①, for medium-①

$$\mu_1 = \frac{c}{v_1} \quad -③$$

$$\Rightarrow \frac{1}{\mu_1} = \frac{v_1}{c} \quad -④$$

Similarly for medium-②

$$\mu_2 = \frac{c}{v_2} \quad -⑤$$

$$\Rightarrow \frac{1}{\mu_2} = \frac{v_2}{c} \quad -⑥$$

where $\mu_1 \rightarrow$ Refractive index of first medium
$\mu_2 \rightarrow$ Refractive index of second medium
$v_1 \rightarrow$ velocity of first medium
$v_2 \rightarrow$ velocity of second medium

When light passes from medium-1 to medium the refractive index of medium-2 with respect medium-1 is written as ${}^{1}\mu_2$ & is called as Relative Refractive index.

→ So Relative Refractive index (${}^{1}\mu_2$) is defined as the ratio between velocity of light in medium-1 to the velocity of light in medium-2

$$\boxed{{}^{1}\mu_2 = \frac{v_1}{v_2}} \quad -⑦$$

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where $v_1 \rightarrow$ velocity of medium 1
$v_2 \rightarrow$ velocity of medium 2

Dividing eqn-⑥ by ⑤

$$\frac{1}{\mu_2} \times \frac{\mu_1}{1} = \frac{v_2}{c} \times \frac{c}{v_1}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

$$\Rightarrow \boxed{\frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}} \quad -⑧$$

From eqn-⑦ & eqn-⑧

$$\boxed{{}^1\mu_2 = \frac{\mu_2}{\mu_1}} \quad -⑨$$

Similarly we can prove that

$$\boxed{{}^2\mu_1 = \frac{\mu_1}{\mu_2}} \quad -⑩$$

$$\Rightarrow \boxed{\frac{1}{{}^2\mu_1} = \frac{\mu_2}{\mu_1}} \quad -⑪$$

From eqn-⑨ & eqn-⑪
it is clear that

$$\boxed{{}^1\mu_2 = \frac{1}{{}^2\mu_1}} \quad -⑫$$

8.4 CRITICAL ANGLE & TOTAL INTERNAL REFLECTION

Consider a source of light 'S' situated in a denser medium say water. Rays starting from S travels from water to air i.e. from denser to rarer medium.

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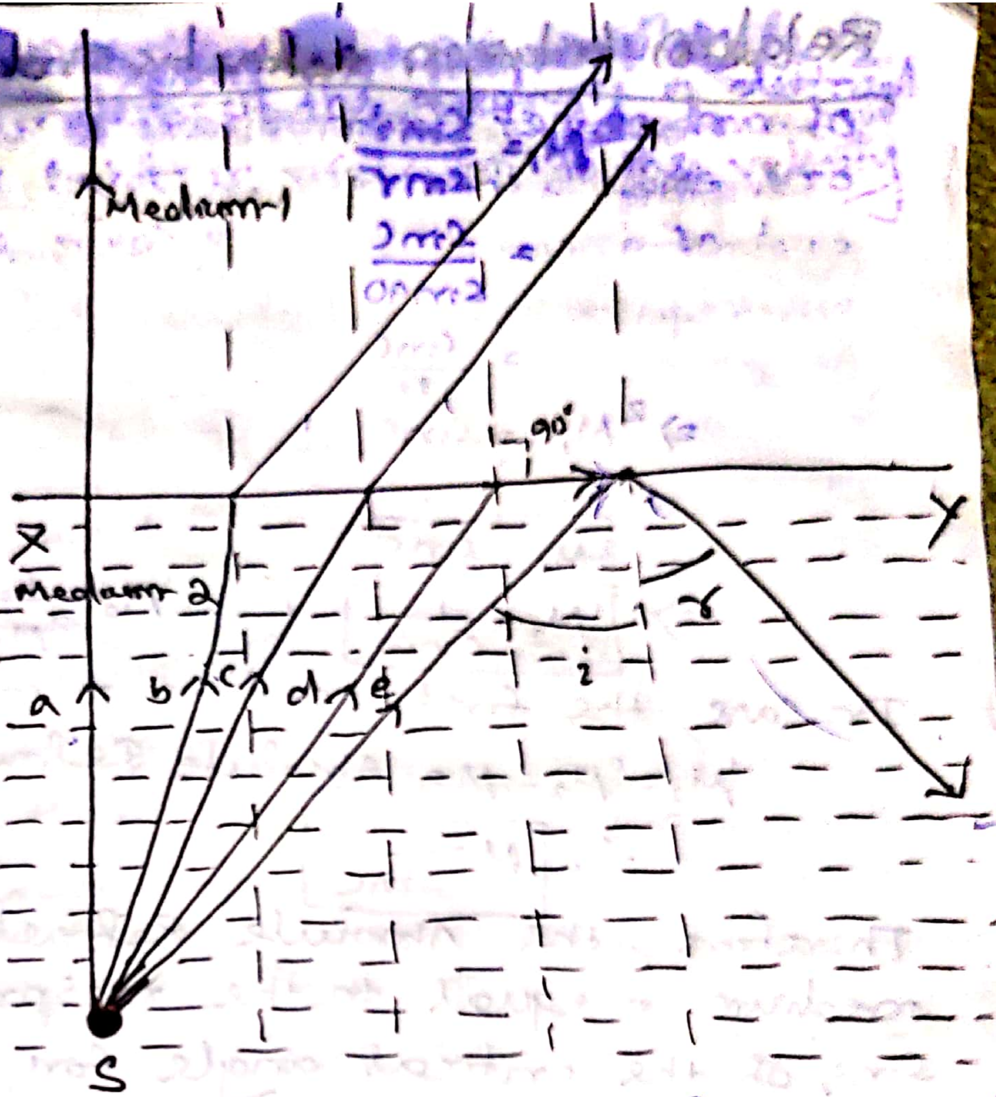
A ray 'a' is incident normally on the interface xy goes undeviated. Ray 'b' and 'c' are incident on the interface at gradually increasing angle of incidence. Therefore, they deviate more & more away from the normal. A ray 'd' is incident at a particular angle of incidence 'c' such that refracted ray is parallel to the surface i.e $r = 90^\circ$. The angle of incidence is called as 'Critical Angle'.



If the angle of incidence of a ray (i) is increases further, it is refracted back into the same medium.

Critical Angle →

It is the angle of incidence of a ray of light in denser medium such that its angle of refraction in the rarer medium is 90°.

Total Internal Reflection →

It is the phenomenon by virtue of which a ray of light travelling from a denser ~~to~~ a rarer medium, is sent back in the same medium provided, it is incident on the interface at an angle greater than critical angle.

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Relation between Refractive Index (μ) & Critical Angle (C)

$${}^2\mu_1 = \frac{\sin i}{\sin r}$$

$$= \frac{\sin C}{\sin 90}$$

$$= \frac{\sin C}{1}$$

$$\Rightarrow {}^2\mu_1 = \sin C$$

or $\frac{1}{{}^2\mu_1} = \frac{1}{\sin C}$

$$\Rightarrow \boxed{{}^1\mu_2 = \frac{1}{\sin C}} \quad \left(\because {}^1\mu_2 = \frac{1}{{}^2\mu_1}\right)$$

In case, the first medium is air or vacuum

${}^1\mu_2 = {}^0\mu_2 = \mu =$ Absolute Refractive Index

$$\therefore \boxed{\mu = \frac{1}{\sin C}}$$

Therefore, the Absolute Refractive Index of medium is equal to the reciprocal of the sine of the critical angle for that medium.

8.5 REFRACTION THROUGH A PRISM

A ray of light suffers two refractions on passing through a prism and hence deviates through a certain angle from its original path.

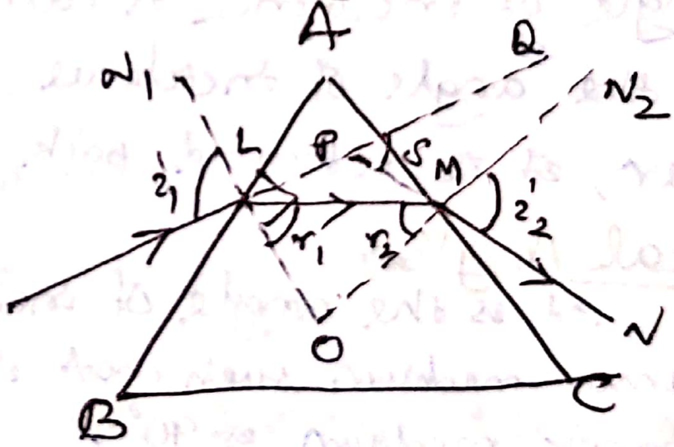

Figure shows ABC as the principal section of a prism with angle of prism as A. A ray of light KL is incident on the face AB of the prism at angle i_1. After refraction it bends towards the normal N_1O along LM at an angle of refraction r_1. Again the refracted ray LM is incident at an angle r_2 on the face AC of the prism. It bends away from the normal N_2O and emerges along MN at angle emergence i_2. In Passing through the Prism, ray KL suffers two refractions and has turned through an angle KQRS, which is the angle of deviation (δ)

$$\boxed{\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}}$$

This relation is called Prism Formula.

where $A \rightarrow$ Angle of Prism

$\delta_m \rightarrow$ Angle of Minimum Deviation.

8.6 FIBRE OPTICS →

→ optical Fibre is a type of cabling technology that uses light to carry voice and data communications over distances both great and small.

→ An optical fibre is a very thin strand of plastic or glass that is used to transmit messages via light. These strands are bundled together in a protective sheath or cover and the whole assembly (the optical fibers and other parts inside the sheath) is referred to as Fiber Optic cable or just fiber.

A ray of light entering the pipe through face 'A' undergoes successive total internal reflections at points P_1, P_2, P_3 --- and emerges out of the face B. AB is called a light pipe, it has allowed light to flow through it from A to B.

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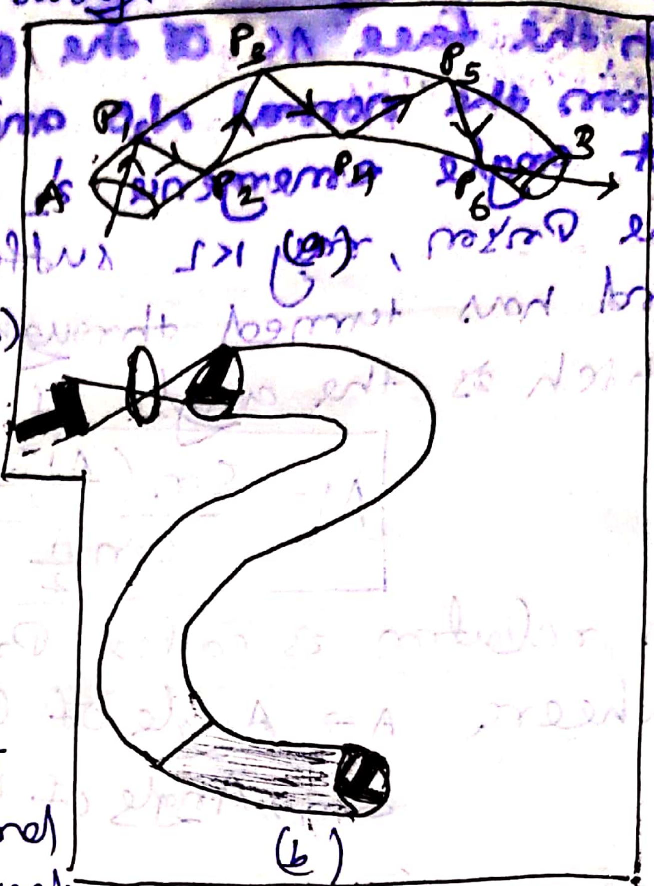
To transmit an image of an object from one end to the other end of a curved path, we require a bundle of pipes (called fibres) packed close to each other. Glass fibres of diameters about 2×10^{-6} m.



Example →

If the inverted image of an object T is focussed on one end of the bundle. An exact production of the image can be obtained at the other end.

Fibre optics has several advantages over traditional metal communication lines.

→ Fibre optic cables have a much greater bandwidth than metal cables. This means that they can carry more data.

→ Fibre optic cables are less susceptible than metal cables to interference.

→ Fibre optic cables are much thinner and lighter than metal wires.

→ Data can be transmitted digitally rather than analogically.

The main disadvantage of fibre optics is that the cables are expensive to install. In addition they are more fragile than wire and are difficult to splice.

PROPERTIES

1. Fibre optic cabling can provide extremely high bandwidths in the range from 100 mbps to 2 gigabits because light has a much higher frequency than electricity.
2. The number of nodes which a fiber optic can support does not depend on its length but on the hub or hubs that connect cables together.
3. Fibre optic cable has much lower attenuation and can carry signal to longer distances without using amplifier and repeaters in between.
4. Fiber optic cable is not affected by EMI effects and can be used in areas where high voltages are passing by.
5. The cost of fiber optic cable is more compared to twisted pair and co-axial.
6. The installation of fiber optic cables is difficult and tedious.

APPLICATIONS :→

1. optical fibres are used for interlinking of computers
2. They are used in decorating flower vases.
3. They are used in military communication because of high degree of data security.
4. They are used for the study of tissues & blood vessels far below the skin.
5. They are used in the field of communication in sending video signals from one place to other.
6. It can be used to transmit high intensity of laser light inside the body for medical purposes.
7. These are used to study the interior of lungs and other parts of body which can be viewed directly otherwise.

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UNIT-9 ELECTROSTATICS & MAGNETOSTATICS

9.1 Electrostatics

Defn →
physics that deals with phenomena due to attractions or repulsions of electric charges but not dependent upon their motion.

Concept →
It is the study of forces between charges, as described by Coulomb's law. We develop the concept of an electric field surrounding charges. We work through examples of the electric field near a line & near a plane & develop formal definations of both electric potential and voltage.

9.2 COULOMB'S LAW IN ELECTROSTATICS

Statement → It states that electro-static force of attraction or repulsion between two charged bodies is directly proportional to the product of their ~~stat~~ charges and varies inversely as the square of the distance between the two bodies.

Explanation →
Suppose two point charges q_1 & q_2 are situated at a distance r from each other

in some medium.

The magnitude of the electrostatic force 'F' which one exerts on the other will be given by

$$F \propto q_1 q_2 \quad -①$$

$$\propto \frac{1}{r^2} \quad -②$$

Combining above eqns, we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \boxed{F = \beta \frac{q_1 q_2}{r^2}} -③$$

where $\beta \rightarrow$ Constant of proportionality. The value of β depends upon the nature of the medium in which the two charges are situated.

Unit

1) In CGS

$$\beta = \frac{1}{K}$$

where K is called the dielectric constant of the medium & taken as K = 1 for free space (vacuum)

$$\boxed{F_1 = \frac{1}{K} \frac{q_1 q_2}{r^2}} \quad \text{(for Any Medium)}$$

$$\boxed{F_2 = \frac{q_1 q_2}{r^2}} \quad \text{(for free space.}$$

2) In SI $\beta = \frac{1}{4\pi \epsilon_0 \epsilon_r}$

where $\epsilon_0 \rightarrow$ Permitivity of free space (vacuum)

9.3 $\epsilon_r \rightarrow$ Relative permitivity of given medium

→ Relative permitivity of a medium is defined as the ratio between absolute permitivity (ϵ) of the medium & the absolute permitivity (ϵ_0) of free space.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}} \text{ or } \boxed{\epsilon = \epsilon_0 \epsilon_r}$$

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In SI,

$$F_1 = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1q_2}{r^2}$$

$$\Rightarrow F_1 = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}$$

But $\epsilon_0 = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$

$$\frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 8.9875 \times 10^9\ Nm^2C^{-2}$$

or $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2C^{-2}$

$$\therefore F_1 = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \boxed{F_1 = \frac{9\times10^9}{\epsilon_r}\frac{q_1q_2}{r^2}}$$ (for any medium)

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$ (for air $\epsilon_r = 1$)

$$\Rightarrow \boxed{F_2 = 9\times10^9\frac{q_1q_2}{r^2}}$$ (for air medium)

It has been observed that ϵ_r has same value as that of K.

$$\boxed{\epsilon_r = K}$$

Unit of ϵ_0

We know $F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} \Rightarrow \boxed{\epsilon_0 = \frac{1}{4\pi}\frac{q_1q_2}{Fr^2}}$

SI →

$$\epsilon_0 = \frac{\text{Coulomb} \times \text{Coulomb}}{\text{Newton} \times (\text{metre})^2}$$

$$= C^2N^{-1}m^{-2}$$

Dimension →

$$\epsilon_0 = \frac{\text{charge}^2}{\text{Force} \times \text{distance}^2} =$$

$$= \frac{[A^1T^1]^2}{[M^1L^1T^{-2}][L^2]}$$

$$= [M^{-1}L^{-3}T^4A^2]$$

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UNIT CHARGE

1) **SI** → $\beta = \frac{1}{4\pi\epsilon_0 K}$

According to coulomb's law, $F = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2}$

For air $K = 1$, $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$$\Rightarrow \boxed{F = 9\times10^9 \frac{q_1 q_2}{r^2}}$$

If $q_1 = q_2 = q$, $r = 1$ metre

& $F = 9\times10^9$ Newton,

then $9\times10^9 = 9\times10^9 \frac{q\times q}{1}$

$\Rightarrow 1 = 1\times q^2$

$\Rightarrow q^2 = 1$

or $\boxed{q = \pm 1 \text{ coulomb}}$

One coulomb of charge is defined as that charge which when placed in air at a distance of 1 metre from an equal and similar charge repels it with a force of 9×10^9 Newton.

1 coulomb = 3×10^9 stat coulomb or e.s.u charge

2) **In CGS** → $\beta = \frac{1}{K}$

According to coulomb's law

$$F = \frac{1}{K} \frac{q_1 q_2}{r^2}$$

For air or vacuum $K = 1$

$$\therefore \boxed{F = \frac{q_1 q_2}{r^2}}$$

If $q_1 = q_2 = q$, $r = 1$ centimeter & $F = 1$ dyne then

Electrostatic unit of charge or stat coulomb is that amount of charge which when placed in air at a distance of 1 cm from a similar charge repels it with a force of 1 dyne.

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9.4 Electric Potential and Electric Potential difference

→ Electric potential →

It is the potential energy of a unit charge in any ~~ch~~ electric field.

or

Electric potential at any point is defined as the -ve of the line integral of electric field from infinity to that point along any path.

$$\boxed{V(\vec{r}) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dl}}$$

→ Electric Potential difference

Electric potential difference between two points in an electric field is defined as the amount of work done in moving unit positive charge from one point to another point against the electric force.

UNIT

$V = \frac{W}{Q}$

SI Unit of potential is Volt

If $W = 1J$, $Q = 1C$

$\therefore V(r) = \frac{1J}{1C} = 1$ volt

So. Potential at any point in an electric field is said to be 1 volt if one Joule of work is done in moving a charge of 1 coulomb between infinity to that point against the electric field along any path.

$$\boxed{1V = 1JC^{-1}}$$

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9.5 Electric Field, Electric Field Intensity (E)

Electric Field → The region around any charge in which its influence can be ~~replaced~~ realised.

Electric Field intensity →

Electric field Intensity at any point is defined as the force experienced by a unit charge placed at that point. The direction of field is given by direction of motion of unit +ve charge if it were free to do so.

Let $\vec{F}(r)$ be the force experienced by a test charge 'q_0' placed at a ~~place~~ point 'p' where the strength of electric field $E(\vec{r})$ is to be calculated. Then

$$\boxed{E(\vec{r}) = \lim_{q_0 \to 0} \frac{F(\vec{r})}{q_0}}$$

UNIT →

9.6 CAPACITY :→

If 'V' is the potential of the conductor due to a charge Q given to it, then

$$Q \propto V$$

or $\boxed{Q = CV}$

where $C \to$ Capacity of the conductor

Thus $\boxed{C = \frac{Q}{V}}$

The Capacity of a conductor is defined as the ratio between the charge on the conductor to its potential.

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If $V = 1$, then $\boxed{C = Q}$

The Capacity of a conductor is also defined as the charge required to raise it through a unit potential.

UNIT

a) SI :- Farad

The Capacity of a conductor is said to be 1 Farad, if a charge of 1 coulomb is sufficient to raise its potential through 1 volt.

$$\boxed{1 \text{ Farad } (F) = \frac{1 \text{ coulomb}}{1 \text{ volt}}}$$

1 microfarad $(\mu F) = 10^{-6} F$

1 micromicrofarad $(\mu\mu F)$ or 1 picofarad $= 10^{-12} F$

b) CGS :→

The Capacity of a conductor is said to be 1 statfarad if a charge of 1 statcoulomb is required to raise its potential through 1 statvolt.

$$\boxed{1 \text{ Statfarad} = \frac{1 \text{ statcoulomb}}{1 \text{ stat volt}}}$$

$$\boxed{1 \text{ farad} = 9 \times 10^{11} \text{ statfarad}}$$

9.7 GROUPING OF CAPACITORS

If a no. of capacitors of capacities $C_1, C_2, \cdots C_n$ are grouped together, the combination behaves like a single capacitor of capacity 'C'. The value of 'C' depends upon the way in which grouping is made.

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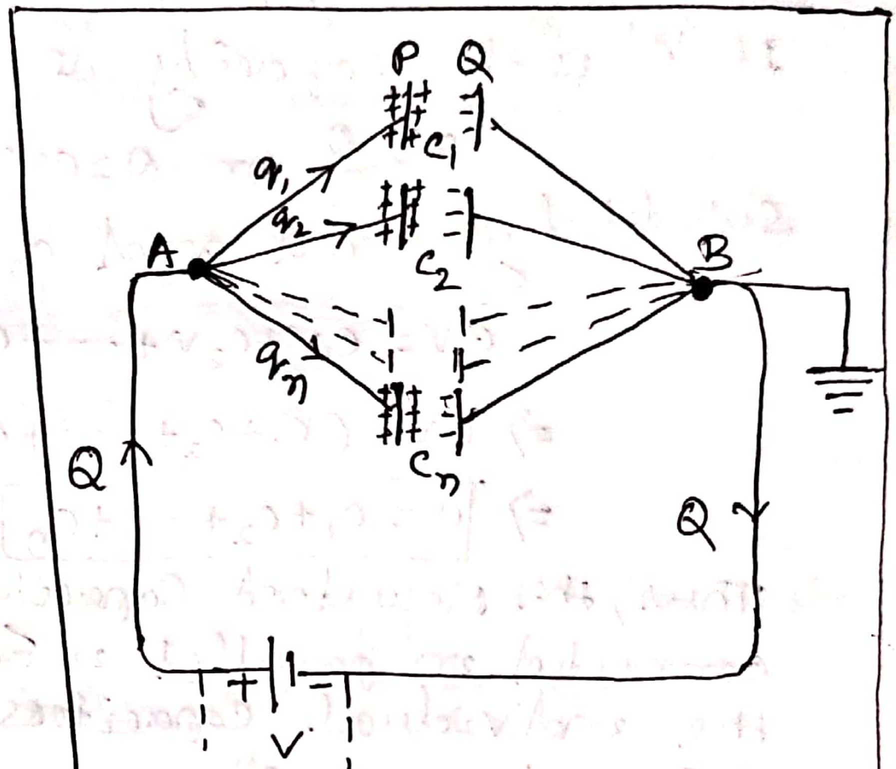
There are two types of grouping are used

(i) Capacitors in Parallel :→

Capacitors are said to be connected in parallel if the first plates (marked P) of all the capacitors are connected together at a point 'A' while the second plates (marked Q) of all the capacitors are connected together at another point 'B'. The point B is also connected to earth while the source of charge is connected between the points A & B.



CAPACITORS IN PARALLEL

Capacitors having capacitors $C_1, C_2 \cdots C_n$ having charges $q_1, q_2, q_3 \cdots q_n$ respectively.

If Q is the total charge, then,

$$Q = q_1 + q_2 + q_3 + \cdots + q_n \quad \text{— (1)}$$

Since all the capacitors are connected between two points A & B, therefore the potential difference across each of them is V.

$$\therefore V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \cdots = \frac{q_n}{C_n}$$

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But $q_1 = C_1 V$

$q_2 = C_2 V$

-- -- --

-- -- --

$q_n = C_n V$

If 'C' is the capacity of the combination, then

$V = \frac{Q}{C}$ or $Q = CV$

Substituting for Q and $q_1, q_2 \ldots q_n$ we get

$CV = C_1 V + C_2 V + \ldots + C_n V$

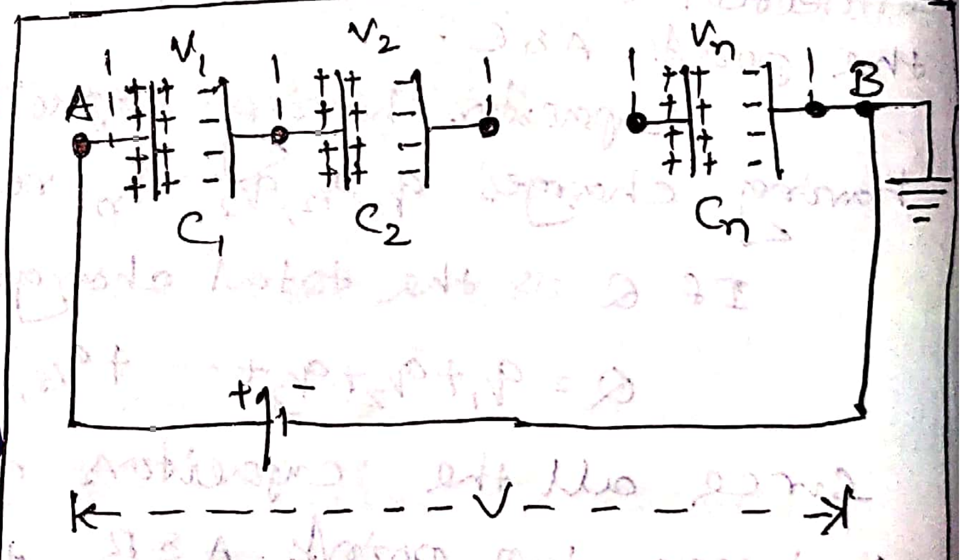
$\Rightarrow CV = (C_1 + C_2 + \ldots + C_n) V$

$\Rightarrow \boxed{C = C_1 + C_2 + \ldots + C_n}$

Thus, the Resultant Capacity of a no. of Capacitors connected in parallel is Equal to the sum of the individual capacitors.

(ii) Capacitors in Series :→

Capacitors are said to be connected in series if the second plate of one is connected with the first plate of the next and so on. This leaves the first plate of first capacitor and the second plate of the last Capacitor as free plates.



Capacitors in Series

These free plates are connected to the terminals A & B. The terminal B is earthed while the source is connected between A and B.

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In a series combination, each capacitor is charged with the same charge while they will be raised through different potentials in accordance with their capacities.

If $V_1, V_2, \dots V_n$ are the potential differences across various capacitors of capacities $C_1, C_2, \dots, C_n$ respectively, then

$$V = V_1 + V_2 + \dots + V_n \quad \text{— (1)}$$

where $V \rightarrow$ potential difference between A&B

Now, $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, $\dots$ $V_n = \frac{q}{C_n}$

Again $V = \frac{q}{C}$

where $C \rightarrow$ Resultant capacity of the combination

Substituting for $V_1, V_2 \dots V_n$ and V in eqn (1), we get

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \dots + \frac{q}{C_n}$$

$$\Rightarrow \frac{q}{C} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}\right]$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

Thus, the reciprocal of the resultant capacity of a number of capacitors, connected in series, is equal to the sum of the reciprocals of their individual capacities.

SIMPLE NUMERICAL PROBLEM

1Q) Find the total capacity when two capacitors of capacities 400 MF and 0.3 mf are connected series

Soln Here $C_1 = 400$ MF

$C_2 = 0.3$ mf $= 0.3 \times 1000$ MF $= 300$ MF

Now $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{400} + \frac{1}{300} = \frac{7}{1200}$ or $C = \frac{1200}{7}$ MF $= 171.44$ MF

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2Q) Three capacitors of 2MF, 3MF and 5MF are connected (i) in series (ii) in parallel. Calculate the resultant capacity of the combination.

Soln $C_1 = 2MF,\ C_2 = 3MF,\ C_3 = 5MF$

(i) When they are series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$$

$$\Rightarrow \frac{1}{C} = \frac{15+10+6}{30}$$

$$\Rightarrow \frac{1}{C} = \frac{31}{30}$$

$$\Rightarrow \boxed{C = \frac{30}{31} MF}$$

(ii) When they are in parallel,

$$C = C_1 + C_2 + C_3$$

$$= 2+3+5$$

$$= 10\ MF$$

$$\Rightarrow \boxed{C = 10^{-5} F}$$

3) a) Three condensers each of 3MF are connected in series with each other. Calculate the resultant capacity of the combination.

Soln Here $C_1 = C_2 = C_3 = 3MF$

When they are connected in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \frac{1}{C} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\Rightarrow \frac{1}{C} = \frac{3}{3} = 1\ MF \quad \Rightarrow \boxed{C = 1\ MF = 10^{-6} F}$$

4Q) Three condensers each of 2MF are connected in parallel with each other. Calculate the net capacity of the combination.

Soln Here $C_1 = C_2 = C_3 = 2MF$

When they are connected in parallel

$$C = C_1 + C_2 + C_3$$

$$= 2+2+2$$

$$\Rightarrow \boxed{C = 6\ MF = 6\times10^{-6} F}$$

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Q) A conductor with a capacitor of 50 MF is connected to a battery of 400 volt. Find the charges on it.

Soln $C = 50\, MF$, $V = 400$ volt

$= 50 \times 10^{-6} F$

$\therefore Q = CV = 50 \times 10^{-6} \times 400 = 0.02 C$

Q) Three condensers are connected in series across a 75 volt supply. The voltage across them are 20, 25 and 30 volts respectively and the charge on each is $3 \times 10^{-3} C$. Find the capacity of each condenser & also of the combination.

Soln $Q = 3 \times 10^{-3} C$, $V = 25 V$, $V_1 = 20V$, $V_2 = 25V$, $V_3 = 30V$.

$\therefore C_1 = \frac{Q}{V_1} = \frac{3 \times 10^{-3}}{20} = 1.5 \times 10^{-4} F$

$C_2 = \frac{Q}{V_2} = \frac{3 \times 10^{-3}}{25} = 1.2 \times 10^{-4} F$

$C_3 = \frac{Q}{V_3} = \frac{3 \times 10^{-3}}{30} = 1 \times 10^{-4} F$

$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$= \frac{1}{1.5 \times 10^{-4}} + \frac{1}{1.2 \times 10^{-4}} + \frac{1}{1 \times 10^{-4}}$

$= \left[\frac{1}{15} + \frac{1}{12} + \frac{1}{10}\right] 10^{5}$

$= \left[\frac{4+5+6}{60}\right] 10^{5}$

$\Rightarrow \frac{1}{C} = \frac{10^5}{4} \Rightarrow \boxed{C = 4 \times 10^{-5} F}$

9.9 COULOMB'S LAW IN MAGNETISM

Statement →

It states that the force of attraction or repulsion between two isolated poles of a magnet varies directly as the product of their pole strengths and inversely proportional to the square of the distance between them.

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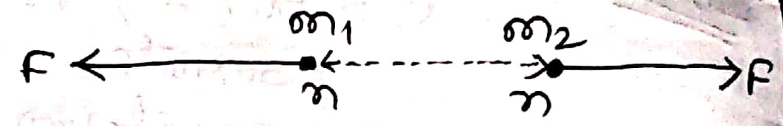
EXPLANATION: Consider two magnetic poles of strength m_1 and m_2 separated by a distance 'r' from each other.



Force between two Magnetic Poles

According this law,

$$F \propto m_1 m_2 \quad -(1)$$

$$F \propto \frac{1}{r^2} \quad -(2)$$

Combining equn (1) & (2)

$$F \propto \frac{m_1 m_2}{r^2} \Rightarrow \boxed{F = K\frac{m_1 m_2}{r^2}} \quad -(3)$$

Where $K \rightarrow$ Proportionality Constant

SI $K = \frac{\mu_0}{4\pi}$

$\therefore$ Equn (3) becomes

$$\boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}} \quad -(4)$$

Where $\mu_0 \rightarrow$ Absolute magnetic Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ WbA^{-1}m^{-1}$$

CGS $K = 1$, $\therefore$ $\boxed{F = \frac{m_1 m_2}{r^2}} \quad -(5)$

UNIT POLE

$\therefore$ Equn (3) becomes

$$F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi}\frac{m_1 m_2}{r^2}$$

$$= 10^{-7}\frac{m_1 m_2}{r^2}$$

If $F = 10^{-7}N$, $m_1 = m_2 = m$ & $r = 1$

then $10^{-7} = 10^{-7}\frac{m \times m}{1}$

$\Rightarrow 1 = \frac{m^2}{1}$

$\Rightarrow m^2 = 1$ or $\boxed{m = \pm 1}$

A Unit Pole in SI system that pole which when placed in air at a distance of 1m from a similar pole repels it with a force of $10^{-7}N$.

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b) CGS system →

From eqn -(5)

$$F = \frac{m_1 m_2}{r^2}$$

If F = 1 dyne, $m_1 = m_2 = m$ & $r = 1$

then $1 = \frac{m^2}{1}$

$\Rightarrow m^2 = 1$

$\Rightarrow \boxed{m = \pm 1}$

So, A unit pole in CGS system is that pole which when placed in air at a distance of 1 cm from a similar pole repels it with a force of 1 dyne.

9.10 MAGNETIC FIELD

Magnetic field, of any magnetic pole, is the region (space) around it in which its magnetic influence can be realised.

MAGNETIC FIELD INTENSITY

Magnetic field intensity, at any point is defined as the force experienced by a unit north pole placed at that point. The direction of field is in the direction in which the unit north pole would move if it were free to do so.

SI

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

If $m_1 = m$, $m_2 = 1$, then $\boxed{F = \frac{\mu_0}{4\pi} \frac{m}{r^2}}$

This gives the magnetic intensity at any point distance r from a magnetic pole of strength m.

CGS

$$F = \frac{m_1 m_2}{r^2}$$

If $m_1 = m$, $m_2 = 1$, then $\boxed{F = \frac{m}{r^2}}$

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If Polarity is south, Magnetic intensity is directed away from it. If Polarity is north, magnetic intensity is directed towards it.

9.11 Magnetic lines of force

Lines of force is the path along which a unit north pole would move if it were free to so.

→ In case of a isolated magnetic poles, the lines of force is a straight line, while in case of a combination of poles (a magnet) it is a curved line.

The arrow head on the lines indicates the direction of motion of the free north pole.

PROPERTIES

(i) Lines of force are directed away from a north pole and are directed towards a south pole. A lines of force starts from a north pole and ends at a south pole if they are isolated poles.

(ii) Tangent, at any point, to the magnetic lines of force, gives the direction of magnetic intensity at the point.

(iii) Two lines of force, never cross each other if the two lines of force were to cross each other, two tangents could be drawn to the line of force at the common point meaning

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thereby two directions of magnetic intensity at that point, which is not possible.

(iv) The lines of force tend to contract longitudinally or lengthwise i.e they possess longitudinal strain. Due to this two unlike poles attract each other.

(v) The lines of force tend to exert lateral (sidewise) pressure i.e they repel each other laterally. This explains the repulsion between two similar poles.

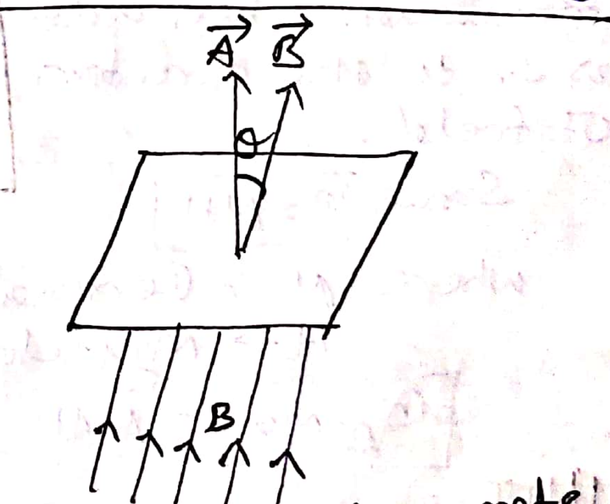
(vi) 4π lines of force start from a unit magnetic poles.

(vii) The no. of lines of force per unit area is proportional to the magnitude of strength of field at that point. Thus more concentration of lines represent stronger magnetic field.

MAGNETIC FLUX (ϕ) →

→ Magnetic flux deals with the study of the no. of lines of force of magnetic field crossing a certain area.

Consider an area 'A' placed in a magnetic field having magnetic induction $\vec{B}$.



Lines of force of magnetic field cutting through area

Let the area be inclined to the direction of 'B' at an angle θ.

Magnetic flux 'ϕ_B' through area 'A' is given by

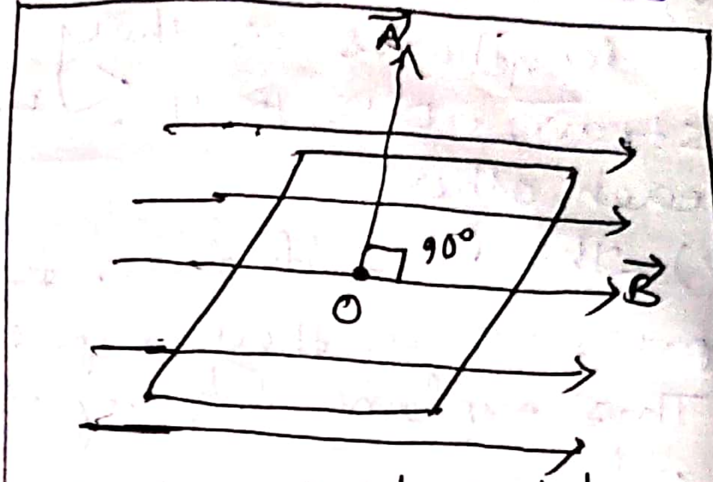
$$\phi_B = \vec{B} \cdot \vec{A}$$
$$= BA\cos\theta$$
$$\Rightarrow \boxed{\phi_B = A(B\cos\theta)}$$

Magnetic flux linked with a surface is defined as the product of area and the component of 'B' perpendicular to the area.

Case-1

If $\theta = 90^\circ$, $\cos\theta = 0^\circ$

when the angle between 'B' and the normal to the surface is 90°. 'B' will be parallel to the surface.



Surface field held parallel to lines of force

$\therefore \phi_B = BA \times 0 = 0$

→ No magnetic flux is linked with the surface when the field is parallel to the surface.

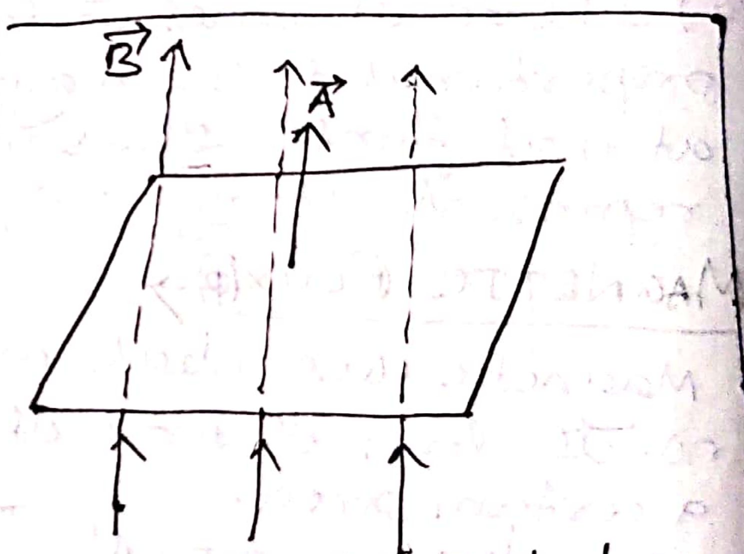
Case-2

If $\theta = 0^\circ$, $\cos\theta = 1$

In case 'B' is ⊥r to surface.

$(\phi_B)_{max} = B \times A \times 1 = BA$

Magnetic flux linked with a surface is maximum when area is ⊥r to the direction of field.



Surface held perpendicular to lines of force

Since $\boxed{B = \mu H}$

where $\mu \rightarrow$ Permeability of the medium

$H \rightarrow$ Magnetic intensity

$$\boxed{(\phi_B)_{max} = \mu A H}$$

UNIT SI. When the area 'A' is ⊥r to the direction of lines of force, magnetic flux linked with it is

$$\phi_B = BA$$

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If $B = 1$ tesla, $A = 1m^2$ then $\phi_B = 1 \times 1 = 1$ weber

Weber → Magnetic flux linked with an area of $1 m^2$ held normal to the direction of lines of force of a magnetic field strength 1 tesla is called 1 weber.

$$\boxed{1 \text{ weber} = 1 \text{ teslametre}^2}$$

CGS Magnetic flux linked with an area of $1 cm^2$ held normal to the direction of lines of force of a magnetic field strength 1 Gauss is called 1 Maxwell.

$$\boxed{1 \text{ Maxwell} = 1 \text{ Gauss} cm^2}$$

Relation between weber & Maxwell

$$1 \text{ Weber} = 1 \text{ tesla} \times 1 m^2$$
$$= 10^4 \text{ gauss} \times (100 cm)^2$$
$$= 10^4 \text{ Gauss} \times 10^4 cm^2$$
$$= 10^8 \text{ Gauss} cm^2$$

$$\therefore \boxed{1 \text{ Weber} = 10^8 \text{ Maxwell}}$$

Dimensions of ϕ_B →

$$\phi_B = BA\cos\theta = \frac{FA}{qv}\cos\theta$$
$$= \frac{[M^1L^1T^{-2}][L^2]}{[A^1T^1][L^1T^{-1}]}$$
$$= [M^1L^2T^{-2}A^{-1}]$$

MAGNETIC FLUX DENSITY (B) →

Magnetic flux density, at any point is defined as the no. of magnetic lines of force passing through a unit area placed at that point of the area is held ⊥r to the direction of lines.

UNIT **SI** ~~weber~~ Tesla

CGS Gauss

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UNIT - 10

CURRENT ELECTRICITY

10.1 ELECTRIC CURRENT

Electric current flowing through a conductor is defined as the rate of flow of charge across any cross-section of the conductor.

If a charge 'q' flows across any cross-section in 't' second, current 'i' is given by

$$\boxed{i = \frac{q}{t}} \quad (1)$$

UNIT

SI Coulomb/sec or ampere

The current flowing through a conductor is said to be one ampere if a charge of 1 coulomb flows across any of its cross-section in 1 second

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

CGS statampere

$$\boxed{1 \text{ amp} = 3 \times 10^9 \text{ statampere}}$$

OHM'S LAW →

The relation between the current flowing through a conductor and the corresponding potential difference across its ends is given by ohm's law which may be stated as

The current is passing through a conductor is directly proportional to potential difference (V) across the ends of the conductor, provided the physical condition of the conductor remains same

$$I \propto V$$

$$\text{or } V \propto I$$

$$\Rightarrow \boxed{V = IR} \quad \Rightarrow \boxed{R = \frac{V}{I}}$$

where R → Resistance of the conductor

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RESISTANCE :→

If 'V' be the potential difference between the two terminals of a conductor and 'i' be the current through it, then

$$\boxed{\frac{V}{i} = R \text{ (constant)}}$$

where R → Resistance of the material

Resistance of a conductor is defined as the ratio between potential difference between the two ends of the conductor to the current flowing through it.

If $i = 1$, $R = V$, then

Resistance of a conductor can be defined as the difference of potential across the two ends of conductor required to pass a unit electric current through it.

Defn → Resistance of any material is defined as the property by virtue of which opposes the flow of electric current through it.

It is denoted by R.

CONCEPT OF RESISTANCE :→

Every conductor contains a large no of free electrons. When a difference of potential is applied between the two ends of the conductor an electric field is set up inside the material of the conductor. A free electron experiences a force, due to this field, which accelerates it from lower to higher potential side. After acquiring some velocity it suffers collision with other free electrons of the material & loses the acquired energy. It again accelerated & goes through the above process repeatedly. Thus, motion

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of the electron can not be termed as free. It experiences resistance forward motion is known as <u>Electrical Resistance</u>.

<u>UNIT</u>

1) <u>SI</u> ohm (Ω)

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ amp}}$$

Resistance of a conductor is said to be 'ohm', if a current of 1 amp flows through it for a potential difference of 1 volt across its end.

2) <u>CGS</u> stat ohm

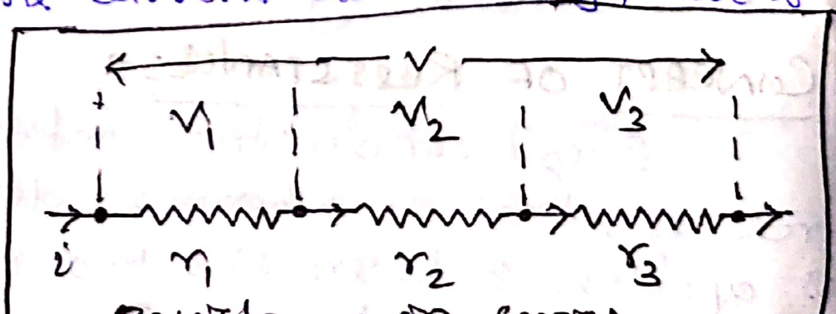
$$1 \text{ statohm} = \frac{1 \text{ stat volt}}{1 \text{ stat amp.}}$$

Resistance of a conductor is said to be 1 statohm, if a current of 1 statamp flows through for a potential difference of 1 statvolt across its ends.

10.3 SERIES AND PARALLEL COMBINATION OF RESISTORS

(i) <u>Resistances in Series</u> →

The resistances are said to be connected in series if same current flows through all of them and there will be distribution of potential which takes place across each resistor.



<u>Resistance in series</u>

Consider resistances r_1, r_2 and r_3 connected in series with each other. Let a current 'i' flows through all of them.

If V_1, V_2 and V_3 are differences of potentials

$V_1 = ir_1$, $V_2 = ir_2$ and $V_3 = ir_3$

across each resistance,

R is the resistance of combination

Total potential difference 'V' across whole of the combination is

$$V = iR$$

Since $V = V_1 + V_2 + V_3$

$$iR = ir_1 + ir_2 + ir_3$$

or $iR = i(r_1 + r_2 + r_3)$

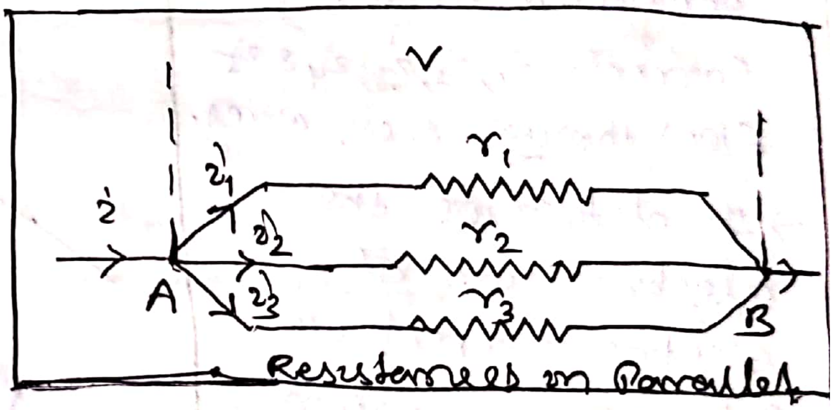
or $\boxed{R = r_1 + r_2 + r_3}$

Thus, if a number of resistances are connected in series with each other, the net resistance of the combination is equal to the sum of their individual resistances.

(ii) Resistances in Parallel →

Resistances are said to be connected in parallel if different currents flow through them.

Consider a number of resistances r_1, r_2 & r_3 connected parallel to each other. A current 'i' is divided into three parts and flows through each of these resistance.



Resistances in Parallel

If V is the difference of potentials across the combination, then

$$V = ir_1 = ir_2 = ir_3$$

~~Since V = V_1 + V_2 + V_3~~

or $i_1 = \frac{V}{r_1}$, $i_2 = \frac{V}{r_2}$, $i_3 = \frac{V}{r_3}$

If R is the resistance of the combination, then $i = \frac{V}{R}$

Since $i = i_1 + i_2 + i_3$

$$\therefore \frac{V}{R} = \frac{V}{r_1} + \frac{V}{r_2} + \frac{V}{r_3}$$

$$\frac{V}{R} = V\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) \Rightarrow \boxed{\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}}$$

Thus, if a number of resistances are connected in parallel, the reciprocal of the resistance of the combination is equal to the sum of the reciprocals of their individual resistances.

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10.4 KIRCHHOFF'S LAWS →

To analyse complicated circuits which contain more than one source of an emf, Kirchhoff's laws are highly convenient.

For the steady current flowing through a circuit, the Kirchhoff's law are applicable.

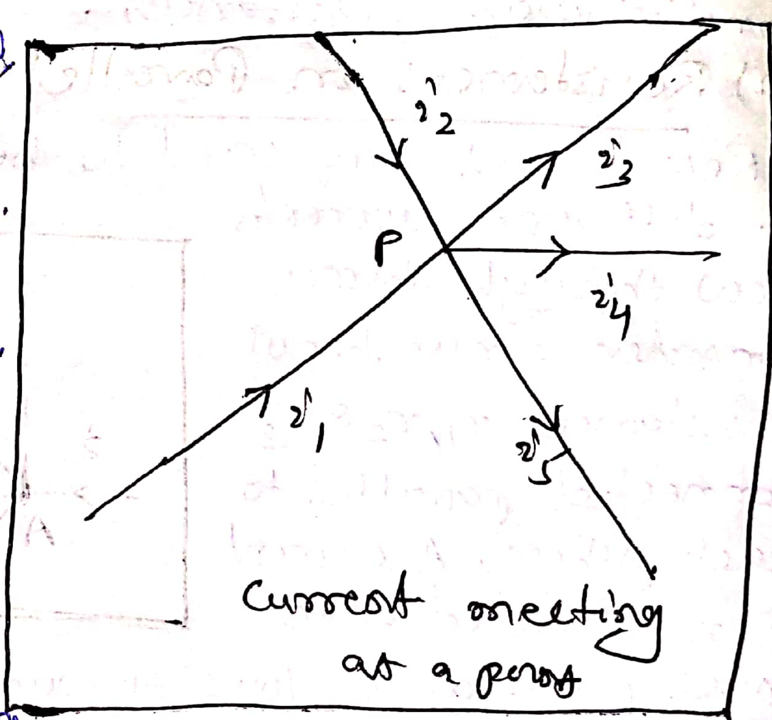
FIRST LAW

It states that algebric sum of currents meeting at a point is zero.

This law is called Kirchhoff's Current law (KCL)

→ To explain this law consider a no. of wires connected at a point P. Currents i_1, i_2, i_3, i_4 & i_5 flow through these wires.

→ To determine the algebric sum of electric currents we follow sign conventions.



(i) The currents approaching a given point are taken as +ve.

(ii) The currents leaving the given point are taken as -ve so that i_1 & i_2 are positives while i_3, i_4 & i_5 are -ve

$$\therefore\ i_1 + i_2 - i_3 - i_4 - i_5 = 0$$

or $\boxed{\Sigma i = 0}$

From eqn (1) $\quad i_1 + i_2 = i_3 + i_4 + i_5$ — (2)

CONCLUSION → The current entering a point must be same as that of leaving it. Hence there can not be any accumulation of charge at any point in a conductor.

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This law is also called as Junction theorem.

SECOND LAW →

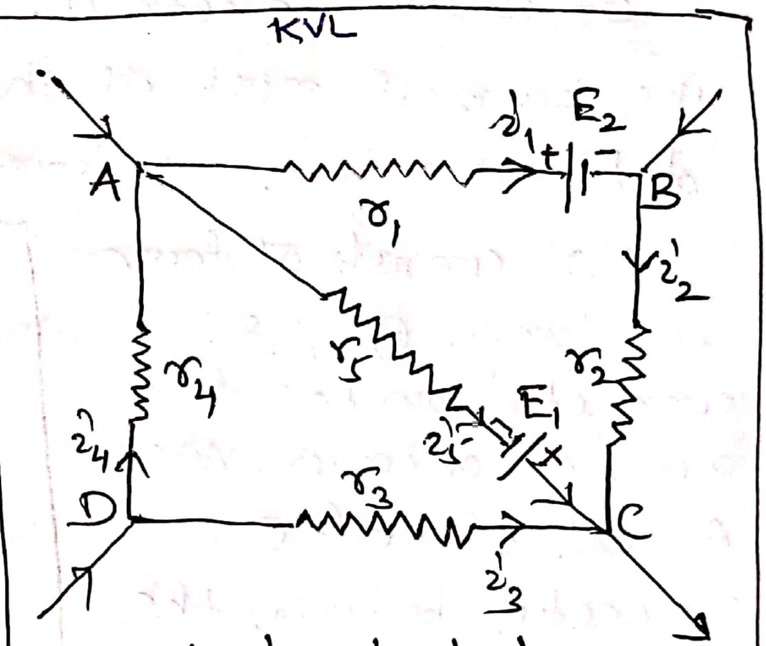
It states that in an closed electric circuit the algebraic sum of emf is equal to the algebraic sum of the products of resistances & the currents flowing through them.

This law is called Kirchhoff's voltage law (KVL)

Figure shows that ~~in~~ an closed electric circuits ~~the algebraic sum of emf is equal to the algebraic sum of the products of resistances~~ ABCD containing resistances r_1, r_2, r_3, r_4 & r_5 in the parts AB, BC, CD, DA & AC respectively.

Let i_1, i_2, i_3, i_4 & i_5 be the respective currents flowing in these parts in the direction shown by arrow heads.

Two sources of emfs E_1 & E_2 are connected in the Mesh.



A closed electric circuit

Inorder to Kirchhoff's voltage we shall follow the following sign conventions.

i) If the electric current flows through the electrolyte of the cell from negative to positive terminal, the emf of the cell is taken as +ve (+E).

ii) If the electric current flows through the electrolyte of the cell from positive to negative terminal, the emf of the cell is taken as -ve (-E).

iii) The current flowing in the clockwise direction is taken as +ve.

iv) The current flowing in the anticlockwise direction is taken as -ve.

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Applying Kirchhoff's 2nd law to the Mesh ABC we can write

$$i_1 r_1 + i_2 r_2 - i_5 r_5 = E_1 - E_2$$

Again applying Kirchhoff's second law to the Mesh ACD, we get

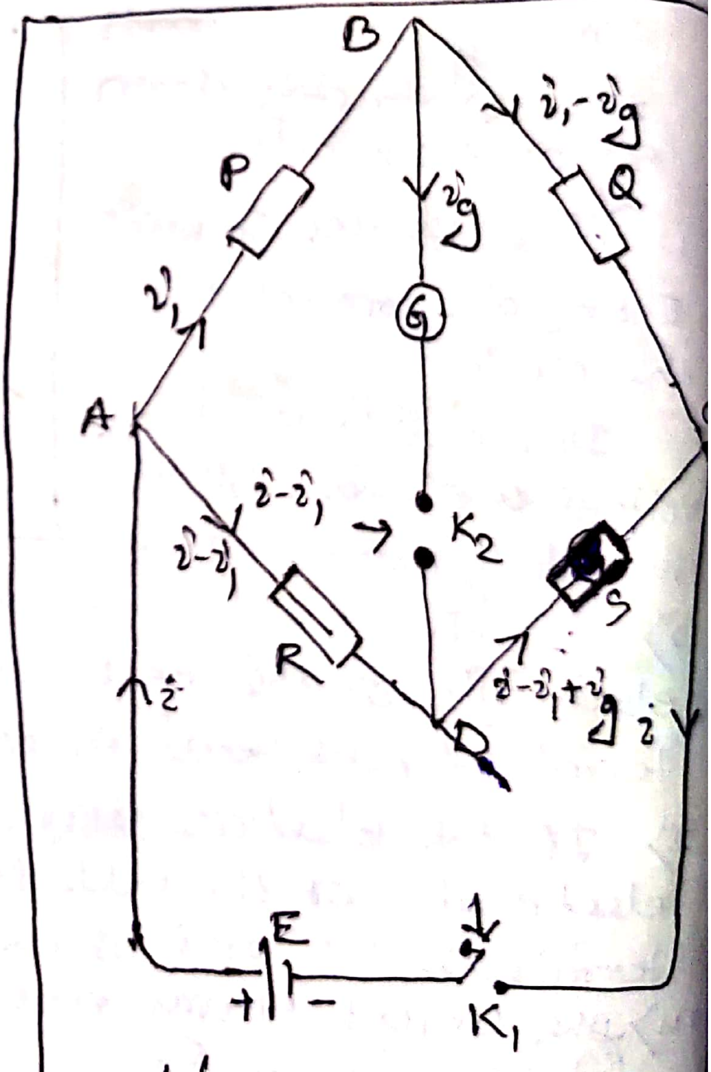
$$i_5 r_5 - i_3 r_3 + i_4 r_4 = E_1$$

The above two equations may be written as in general form $\boxed{\Sigma ir = \Sigma E}$

10.5 WHEATSTONE BRIDGE →

It is an electrical arrangement which forms the basis of most of the instruments used to determine an unknown resistance.

It consists of four resistances P, Q, R & S connected in the four arms of a square ABCD. A cell of emf E is connected between the points A & C through a one way key K_1. A Sensitive Galvanometer of resistance G is connected between the terminals B and D through another one way key K_2.



WHEATSTONE BRIDGE

After closing the keys K_1 & K_2, the resistances P, Q, R & S are so adjusted that the galvanometer shows no deflection. In this position the wheatstone bridge is said to be balanced.

Applying Kirchhoff's current law, the distribution of current and their directions through various resistances are as follows.

Now giving +ve sign to the current flowing in clockwise direction & negative sign to the currents flowing in anticlockwise directions and applying Kirchhoff's voltage law to the mesh ABD, we can write

$$i_1 P + i_g G - (i - i_1) R = 0 \quad \text{--- (1)}$$

Similarly applying Kirchhoff's 2nd law to Mesh BCD, we can write

$$(i_1 - i_g) Q - (i - i_1 + i_g) S - i_g \cdot G = 0 \quad \text{--- (2)}$$

The RHS of both equations (1) & (2) are zero because there is no source of emf in both the closed circuits ABD & BCD.

Since the Bridge is balanced, therefore the current 'i_g' flowing through the arm DB is zero.

Putting $i_g = 0$ in eqn -(1) & (2), we get

$$i_1 P - (i - i_1) R = 0 \Rightarrow i_1 P = (i - i_1) R \quad \text{--- (3)}$$

and $\quad i_1 Q - (i - i_1) S = 0 \Rightarrow i_1 Q = (i - i_1) S \quad \text{--- (4)}$

Dividing eqn (3) by (4), we get

$$\frac{i_1 P}{i_1 Q} = \frac{(i - i_1) R}{(i - i_1) S}$$

or $\boxed{\frac{P}{Q} = \frac{R}{S}}$

This is the required condition for the bridge to be balanced and gives the principle of Wheatstone Bridge.

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SIMPLE PROBLEMS

1) Q) Two coils have a combined resistance of 10 ohms when in series and 2.4 ohm when in Parallel. Find the resistance of each coil.

Soln Let r_1 & r_2 are the resistance of two coils

$$r_1 + r_2 = 10 \quad \text{— (1)}$$

$$\Rightarrow \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2.4}$$

$$\text{or} \quad \frac{r_1 r_2}{r_1 + r_2} = 2.4 \quad \text{— (2)}$$

Substituting for $r_1 + r_2$ from eqn (1) & (2)

$$\frac{r_1 r_2}{10} = 2.4$$

$$\therefore r_1 r_2 = 24 \quad \text{— (3)}$$

From eqn (1) $\quad r_2 = 10 - r_1$

Substituting in eqn (ii),

$$r_1 (10 - r_1) = 24$$

$$\Rightarrow 10 r_1 - r_1^2 = 24$$

$$\Rightarrow r_1^2 - 10 r_1 + 24 = 0$$

$$\Rightarrow (r_1 - 6)(r_1 - 4) = 0$$

Either

$r_1 - 6 = 0$	or $r_1 - 4 = 0$
$\therefore r_1 = 6\,\Omega$	$\therefore r_1 = 4\,\Omega$
$\therefore r_2 = 10 - r_1 = 10 - 6$	$r_2 = 10 - r_1$
$\Rightarrow r_2 = 4\,\Omega$	$\therefore r_2 = 10 - 4 = 6\,\Omega$
	$\Rightarrow r_2 = 6\,\Omega$

Thus the two coils have resistances 6Ω & 4Ω respectively.

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UNIT-11 ELECTROMAGNETISM & ELECTROMAGNETIC INDUCTION

11.1 ELECTROMAGNETISM

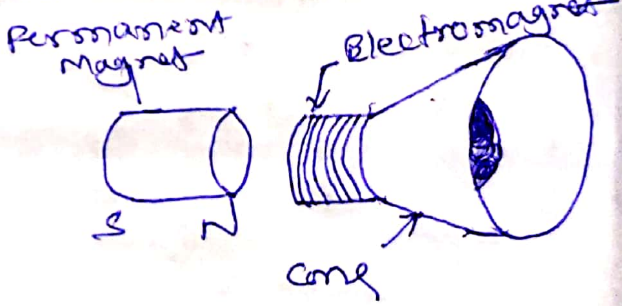
The branch of physics that deals with electricity and magnetism, as ~~ex~~ when an electric current or a changing electric field generates a magnetic field or when a changing magnetic field generates an electric field.

→ It is a process where a magnetic field is created by introducing the current in the conductor.

Example

In order to convert electrical waves into audible sound, the speakers are designed.

A metal coil is attached to a permanent magnet and when current passes through the coil it generates a magnetic field.



The newly formed magnetic field is repelled by the permanent magnetic field resulting in vibrations. These vibrations are amplified by the cone like structure causing the sound. This is how speakers work based on electromagnetism.

11.2 FORCE ACTING ON A CURRENT CARRYING CONDUCTOR PLACED IN A UNIFORM MAGNETIC FIELD

Conductor has free electrons in it. When a potential difference is maintained across the two ends of the conductor, the electrons drift from lower potential to higher potential with a

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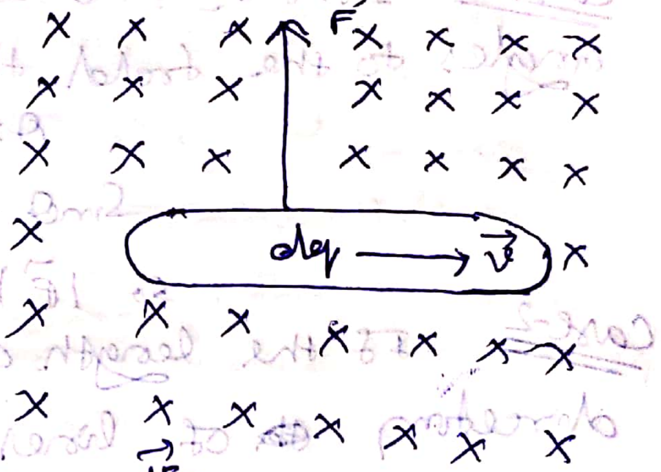
small velocity. These electrons constitute a current through the conductor. When the electrons move in a magnetic field, they experiences a force $\vec{F}$.

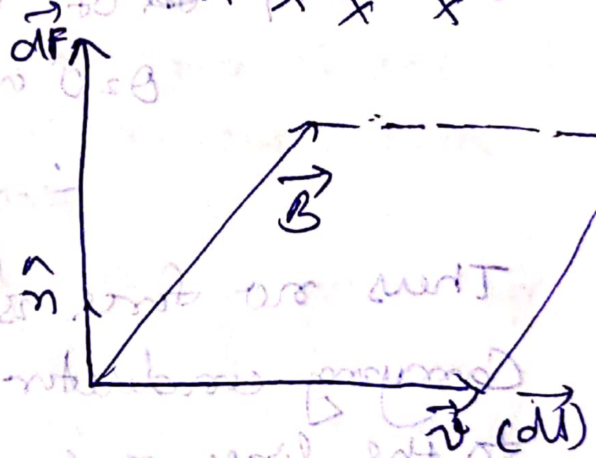
Consider a conductor xy placed in a uniform magnetic field $\vec{B}$ acting inwards at right angles to the plane of paper. Let a current i flow through the conductor from x to y.

Let dq be a small amount of +ve charge moving from x to y with a velocity 'v'. The Force $\vec{dF}$ experienced by this charge is given by



$$\vec{dF} = dq(\vec{v}\times\vec{B})$$

If the charge travels a distance $\vec{dl}$ in time dt, then $\vec{v} = \frac{\vec{dl}}{dt}$



$$\vec{dF} = dq \times \left(\frac{\vec{dl}}{dt}\times\vec{B}\right)$$

$$= \frac{dq}{dt}\left(\vec{dl}\times\vec{B}\right)$$

$$\Rightarrow \vec{dF} = i\left(\vec{dl}\times\vec{B}\right) \quad \text{— (1)}$$

Applying the rule of cross-product, it can be seen that force $\vec{dF}$ acts vertically upwards as shown.

Net force $\vec{F}$ acting on the conductor can be obtained by integrating equation

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$\therefore \vec{F} = \int dF$

$= i\int \vec{dl} \times \vec{B}$

$= i\int dl\, B \sin\theta\, \hat{n}$

$\vec{F} = ilB\sin\theta\, \hat{n}$

where $\hat{n}$ is a unit vector in a direction perpendicular to the plane containing $\vec{l}$ & $\vec{B}$. θ is the angle between $\vec{l}$ & $\vec{B}$.

Case-1 If the conductor placed at right angles to the field then

$\theta = 90^\circ$

$\therefore \sin\theta = 1$

So $|\vec{F}| = ilB$ (maximum)

Case-2 If the length of conductor is along the direction of lines of force then

$\theta = 0^\circ$ or $\theta = 180^\circ$

$\therefore \sin\theta = 0$

$|F| = 0$

Thus no force is experienced by current carrying conductor when its length is parallel to the lines of force of magnetic field whatever be the direction of electric current.

FLEMING'S LEFT HAND RULE →

Whenever a current carrying conductor comes under a magnetic field, there will be a force acting on the conductor. The direction of this force can be found by using Fleming's Left Hand rule.

This rule states that " stretch first finger, central ~~dog~~ finger and thumb of your left hand in mutually perpendicular directions, If the first finger points towards the magnetic field, central finger points towards electric current, then the thumb gives the direction of force acting on the conductor.

11.3 FARADAY'S LAW OF ELECTRO-MAGNETIC INDUCTION

This law deals with the induction of an emf in an electric circuit when magnetic flux linked with the circuit changes. They are

(i) whenever a magnetic flux linked with the circuit changes, an emf is induced in it.

(ii) the induced emf exists in the circuit so long as the change in magnetic flux linked with it continues.

(iii) the induced emf is directly proportional to the -ve rate of change of magnetic flux linked with it ~~continues~~ circuit.

If $d\phi_B$ is the change in magnetic flux linked with a circuit that takes place in a time dt, then the rate of change of magnetic flux $= \frac{d\phi_B}{dt}$

If E is the emf induced in the circuit as a result of this change.

$$E \propto \frac{-d\phi_B}{dt}$$

$$\text{or } E = -K\frac{d\phi_B}{dt}$$

If $K=1$, then $\boxed{E = \frac{-d\phi_B}{dt}}$

-ve sign is due to direction of induced emf.

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11.4 LENZ'S LAW →

It deals with the direction of emf induced in the circuit due to a change in magnetic flux linked with it.

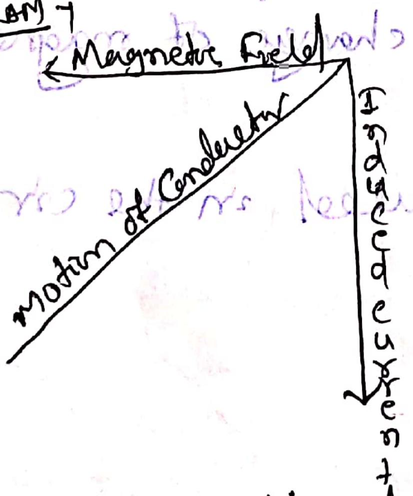
It states that direction of induced emf is such that it tends to oppose the change which produces it.

11.5. FLEMING'S RIGHT HAND RULE →

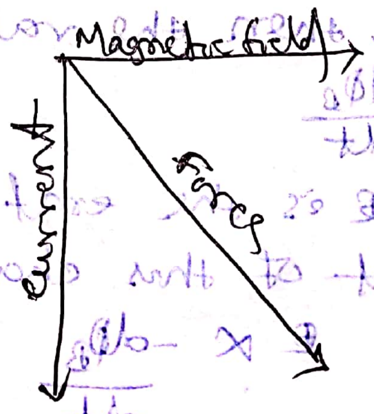
It is the rule to find out the direction of induced current in a conductor.

This rule states that stretch first finger, central finger and thumb of your right hand in three mutually perpendicular directions. If the first finger points towards the direction ~~of induced or generated current within the conductor~~ magnetic field. The thumb points towards the direction of motion of conductor, the direction of central finger points the direction of induced or generated current within the conductor.

DIAGRAM →



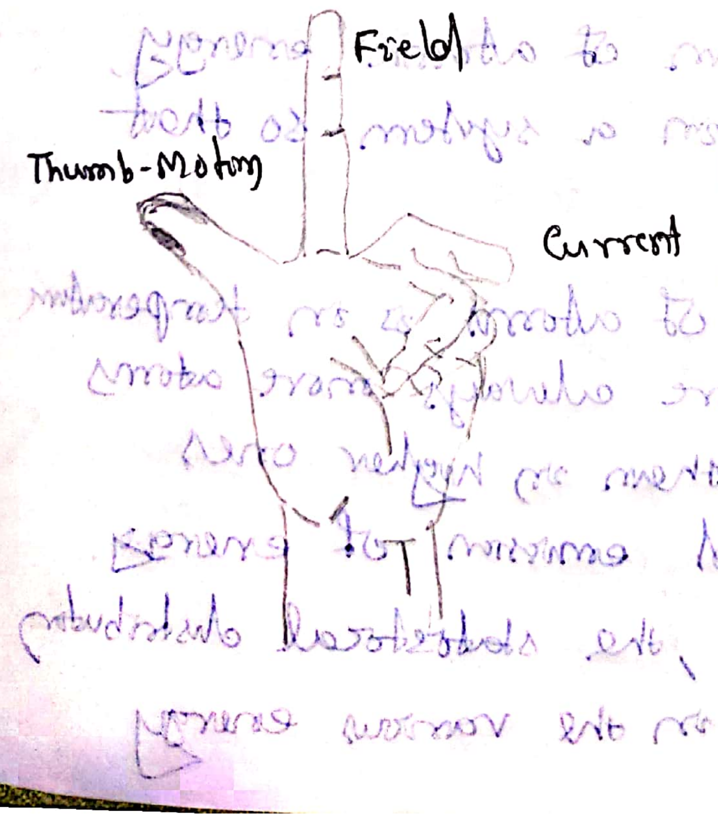
(Fleming's Right Hand rule)

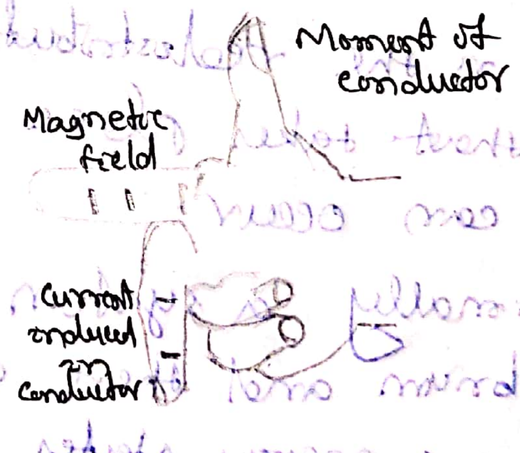


(Fleming's Left Hand Rule)

11.6 COMPARISION BETWEEN FLEMING's RIGHT HAND RULE & FLEMING's LEFT HAND RULE

Fleming's left Hand Rule	Fleming's Right Hand Rule
1. This rule used to determine the direction of magnetic force.	1. This rule is used to determine the direction of induced current.
2. central finger signifies for electric current.	2. central finger signifies induced or generated current within the conductor.
3. Thumb gives the direction of the magnetic force.	3. Thumb points in the direction of motion of conductor
Application	**Application**
1. The direction of the magnetic field inside the solenoid is determined by Right-Hand thumb rule.	1. The direction of induced current within the electric generator is determined by Fleming's Right Hand rule.
1. It is used for electric Motors	2. It is used for electric generators





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UNIT-12 MODERN PHYSICS

12.1 LASER & Laser Beam

LASER → It is a device to produce a powerful monochromatic, polarised beam of light waves which are identical in nature & can travel over long distances without any change of their intensity.

→ The term LASER stands for "Light Amplification by Stimulated Emission of Radiation".

LASER BEAM →

It is extremely intense, coherent and highly parallel beam of light

12.2 PRINCIPLE OF LASER

POPULATION INVERSION →

→ Population inversion occurs while as a system (such as a group of atoms or molecules) exist in a state which more members of the system are in higher excited states than in the lower unexcited state.

→ It is the redistribution of atomic energy levels that takes place in a system so that laser can occur.

Normally a system of atoms is in temperature equilibrium and there are always more atoms in lower energy states than in higher ones. Although absorption and emission of energy is a continuous process, the statistical distribution (population) of atoms in the various energy

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states is constant. when the distribution is disturbed by pumping energy into a system, a population inversion will take place in which more atoms will exist in the higher energy states than in the lower.

OPTICAL PUMPING →

It is a process in which light is used to raise (or pump) electrons from a lower energy level in an atom or molecule to a higher one.

It is commonly used in laser construction, to pump the active laser medium so as to achieve population inversion.

12.3 PROPERTIES OF LASER

i) DIRECTIONALITY → It is unidirectional and convergent beam.

ii) MONOCHROMATIC → It is more or less single wavelength.

iii) BRIGHTNESS → It is high energentic. Pulse energy is high 10^{17} Joule.

iv) INTENSITY → As it has the ability to focus over as small an area as 10^{-6} cm, therefore its is highly intense.

v) COHERENT → It is highly coherent. It has wide range of wavelength and frequency.

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APPLICATIONS OF LASER →

(1) It will be a useful tool in the study of distant planets.

(2) It is used for automatically guided Rockets and satellites.

(3) In optical fibre we use laser beam to study the interior part of the ~~body~~ body which can't be viewed directly.

(4) It is used in surgery, radar, welding microscope wires and vapourising the known material

(5) It is used to decompose noxious substances from industrial waste to convert them to harmless substances

(6) Laser beams are capable of destroying enemy war planes.

(7) A laser gun can kill human being without any sound

(8) Due to high monochromaticity and high frequency it can help in sending hundreds of message at a time on radio, television and telescope.

12.4 WIRELESS TRANSMISSION
(Ground Waves, Sky Waves, Space Waves)

Wireless communication involves no physical link established between two or more devices communicating wirelessly.

Wireless signals are spread over in the air and are received and interpreted by appropriate antennas.

Radio waves form an important tools for communicating message from one place to other. propagation of radio wave can take place in any of the three methods.

(1) Ground Waves →

These waves propagate over the earth's surface in low and medium frequencies. These are mainly used for transmission between the surface of earth and the ionosphere.

The Radio waves emitted by a transmitter travel in a straight line. As such these are not able to reach distant point due to the curvature of earth. The station situated close to the transmitter can catch these rays directly called Ground waves.

Due to their absorption by ground, the signals are weaker & their absorption increases with an increase in frequency of wave.

Application →

1. These waves can be used for one-way communication from the military to submerged submarines as they penetrate to a significant depth into seawater.

2. AM, FM & television broadcasting can be done with the help of ground waves.

(ii) Sky Waves →

The stations, which become inaccessible to ground waves due to the curvature of earth, can receive waves after reflection from the ionosphere. These waves are called sky waves.

It is either reflected or refracted back waves to the earth from the ionosphere which is an electrically charged layer of the upper atmosphere.

Medium and shortwave frequencies can be refracted to earth which is beyond the horizon which makes them useful in the transcontinental transmission of the waves.

Application →

1. Satellite communications takes place with the help of sky wave propagation as it is dependent on the upper atmospheric conditions.
2. Mobile transmissions.

Hence, the waves with a frequency range from 2 MHz to 30 MHz can be propagated by this method.

iii) Space Wave Propagation →

The electromagnetic waves emitted by transmitter antenna travel directly from the transmitting antenna to the receiving antenna, are called space waves and this type of propagation is called space wave propagation. Thus, their propagation takes place in between two highly placed antennae.

High frequency electromagnetic waves can not be transmitted as ground waves due to high energy losses. These waves are absorbed by the ionosphere hence they can not be transmitted via skywave propagation. Therefore such high-frequency electromagnetic waves are directly transmitted through earth atmosphere using a transmitting antenna. As these waves travel in a straight line, the receiving antenna must be in the line of sight of the transmitting antenna.

The TV signals having frequency band 54-806 MHz can propagate neither via ground waves (due to high absorption in the atmosphere) nor via sky waves (due to non-reflection of the ionosphere). Hence TV signals can only be propagated through space wave only.

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